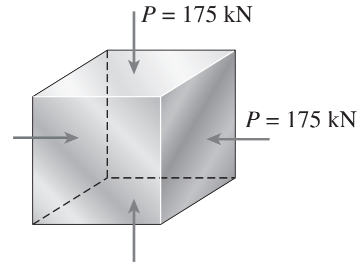
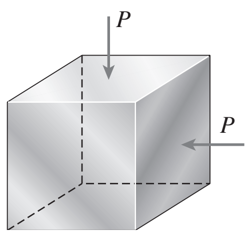
Problem 7.5-8 A brass cube 50 mm on each edge is compressed in two perpendicular directions by forces $P = 175$ kN (see figure).

Calculate the change ΔV in the volume of the cube and the strain energy U stored in the cube, assuming $E = 100$ GPa and $\nu = 0.34$.



Solution 7.5-8 **Biaxial stress-cube**



Side $b = 50$ mm $\quad P = 175$ kN
$E = 100$ GPa $\quad \nu = 0.34$ (Brass)

$$\sigma_x = \sigma_y = -\frac{P}{b^2} = -\frac{(175\text{ kN})}{(50\text{ mm})^2} = -70.0\text{ MPa}$$

CHANGE IN VOLUME

$$\text{Eq. (7-47): } e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y) = -448 \times 10^{-6}$$

$$V_0 = b^3 = (50\text{ mm})^3 = 125 \times 10^3\text{ mm}^3$$

$$\Delta V = eV_0 = -56\text{ mm}^3 \longleftarrow$$

(Decrease in volume)

STRAIN ENERGY

$$\text{Eq. (7-50): } u = \frac{1}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y)$$

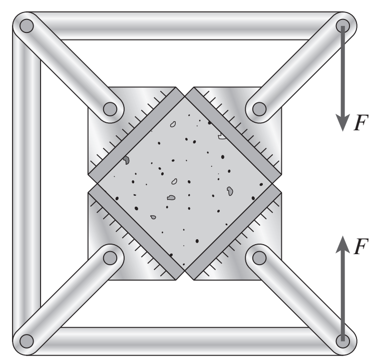
$$= 0.03234\text{ MPa}$$

$$U = uV_0 = (0.03234\text{ MPa})(125 \times 10^3\text{ mm}^3)$$

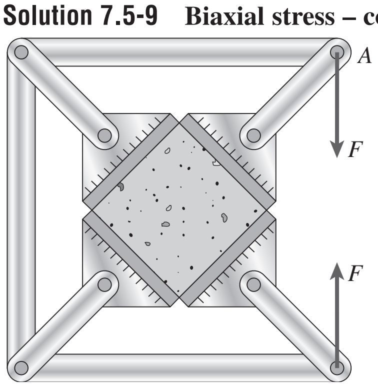
$$= 4.04\text{ J} \longleftarrow$$

Problem 7.5-9 A 4.0-inch cube of concrete ($E = 3.0 \times 10^6$ psi, $\nu = 0.1$) is compressed in *biaxial stress* by means of a framework that is loaded as shown in the figure.

Assuming that each load F equals 20 k, determine the change ΔV in the volume of the cube and the strain energy U stored in the cube.

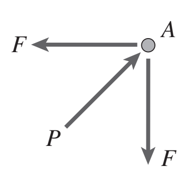


Solution 7.5-9 **Biaxial stress – concrete cube**



$b = 4$ in.
$E = 3.0 \times 10^6$ psi
$\nu = 0.1$
$F = 20$ kips

Joint A:
$P = F\sqrt{2}$
$= 28.28$ kips



$$\sigma_x = \sigma_y = -\frac{P}{b^2} = -1768\text{ psi}$$

CHANGE IN VOLUME

$$\text{Eq. (7-47): } e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y) = -0.0009429$$

$$V_0 = b^3 = (4\text{ in.})^3 = 64\text{ in.}^3$$

$$\Delta V = eV_0 = -0.0603\text{ in.}^3 \longleftarrow$$

(Decrease in volume)

STRAIN ENERGY

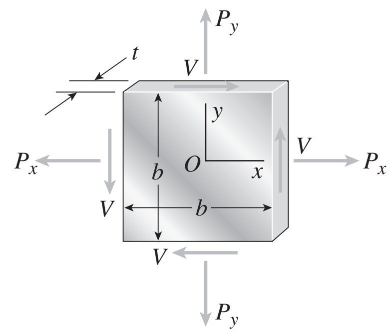
$$\text{Eq. (7-50): } u = \frac{1}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y)$$

$$= 0.9377\text{ psi}$$

$$U = uV_0 = 60.0\text{ in.-lb} \longleftarrow$$

Problem 7.5-10 A square plate of width b and thickness t is loaded by normal forces P_x and P_y, and by shear forces V, as shown in the figure. These forces produce uniformly distributed stresses acting on the side faces of the plate.

Calculate the change ΔV in the volume of the plate and the strain energy U stored in the plate if the dimensions are $b = 600$ mm and $t = 40$ mm, the plate is made of magnesium with $E = 45$ GPa and $\nu = 0.35$, and the forces are $P_x = 480$ kN, $P_y = 180$ kN, and $V = 120$ kN.



Probs. 7.5-10 and 7.5-11

Solution 7.5-10 **Square plate in plane stress**

$b = 600$ mm $\quad t = 40$ mm

$E = 45$ GPa $\quad \nu = 0.35$ (magnesium)

$P_x = 480$ kN $\quad \sigma_x = \dfrac{P_x}{bt} = 20.0$ MPa

$P_y = 180$ kN $\quad \sigma_y = \dfrac{P_y}{bt} = 7.5$ MPa

$V = 120$ kN $\quad \tau_{xy} = \dfrac{V}{bt} = 5.0$ MPa

CHANGE IN VOLUME

Eq. (7-47): $e = \dfrac{1 - 2\nu}{E}(\sigma_x + \sigma_y) = 183.33 \times 10^{-6}$

$V_0 = b^2 t = 14.4 \times 10^6$ mm^3

$\Delta V = eV_0 = 2640$ mm^3 ←

(Increase in volume)

STRAIN ENERGY

Eq. (7-50): $u = \dfrac{1}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y) + \dfrac{\tau_{xy}^2}{2G}$

$G = \dfrac{E}{2(1 + \nu)} = 16.667$ GPa

Substitute numerical values:

$u = 4653$ Pa

$U = uV_0 = 67.0$ N · m $= 67.0$ J ←

Problem 7.5-11 Solve the preceding problem for an aluminum plate with $b = 12$ in., $t = 1.0$ in., $E = 10{,}600$ ksi, $\nu = 0.33$, $P_x = 90$ k, $P_y = 20$ k, and $V = 15$ k.

Solution 7.5-11 **Square plate in plane stress**

$b = 12.0$ in. $\quad t = 1.0$ in.

$E = 10{,}600$ ksi $\quad \nu = 0.33$ (aluminum)

$P_x = 90$ k $\quad \sigma_x = \dfrac{P_x}{bt} = 7500$ psi

$P_y = 20$ k $\quad \sigma_y = \dfrac{P_y}{bt} = 1667$ psi

$V = 15$ k $\quad \tau_{xy} = \dfrac{V}{bt} = 1250$ psi

CHANGE IN VOLUME

Eq. (7-47): $e = \dfrac{1 - 2\nu}{E}(\sigma_x + \sigma_y) = 294 \times 10^{-6}$

$V_0 = b^2 t = 144$ in.^{3}

$\Delta V = eV_0 = 0.0423$ in.^{3} ←

(Increase in volume)

STRAIN ENERGY

Eq. (7-50): $u = \dfrac{1}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y) + \dfrac{\tau_{xy}^2}{2G}$

$G = \dfrac{E}{2(1 + \nu)} = 3985$ ksi

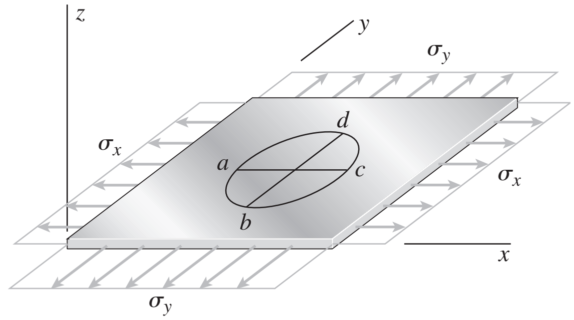
Substitute numerical values:

$u = 2.591$ psi

$U = uV_0 = 373$ in.-lb ←

Problem 7.5-12 A circle of diameter $d = 200$ mm is etched on a brass plate (see figure). The plate has dimensions $400 \times 400 \times 20$ mm. Forces are applied to the plate, producing uniformly distributed normal stresses $\sigma_x = 42$ MPa and $\sigma_y = 14$ MPa.

Calculate the following quantities: (a) the change in length Δac of diameter ac; (b) the change in length Δbd of diameter bd; (c) the change Δt in the thickness of the plate; (d) the change ΔV in the volume of the plate, and (e) the strain energy U stored in the plate. (Assume $E = 100$ GPa and $\nu = 0.34$.)



Solution 7.5-12 Plate in biaxial stress

$\sigma_x = 42$ MPa $\quad \sigma_y = 14$ MPa
Dimensions: $400 \times 400 \times 20$ (mm)
Diameter of circle: $d = 200$ mm
$E = 100$ GPa $\quad \nu = 0.34$ (Brass)

(a) CHANGE IN LENGTH OF DIAMETER IN x DIRECTION

Eq. (7-39a): $\varepsilon_x = \dfrac{1}{E}(\sigma_x - \nu\sigma_y) = 372.4 \times 10^{-6}$

$\Delta ac = \varepsilon_x d = 0.0745$ mm ⟵ (increase)

(b) CHANGE IN LENGTH OF DIAMETER IN y DIRECTION

Eq. (7-39b): $\varepsilon_y = \dfrac{1}{E}(\sigma_y - \nu\sigma_x) = -2.80 \times 10^{-6}$

$\Delta bd = \varepsilon_y d = -560 \times 10^{-6}$ mm ⟵ (decrease)

(c) CHANGE IN THICKNESS

Eq. (7-39c): $\varepsilon_z = -\dfrac{\nu}{E}(\sigma_x + \sigma_y) = -190.4 \times 10^{-6}$

$\Delta t = \varepsilon_z t = -0.00381$ mm ⟵ (decrease)

(d) CHANGE IN VOLUME

Eq. (7-47): $e = \dfrac{1 - 2\nu}{E}(\sigma_x + \sigma_y) = 179.2 \times 10^{-6}$

$V_0 = (400)(400)(20) = 3.2 \times 10^6\ \text{mm}^3$

$\Delta V = eV_0 = 573\ \text{mm}^3$ ⟵ (increase)

(e) STRAIN ENERGY

Eq. (7-50): $u = \dfrac{1}{2E}(\sigma_x^2 + \sigma_y^2 - 2\nu\sigma_x\sigma_y)$

$= 7.801 \times 10^{-3}$ MPa

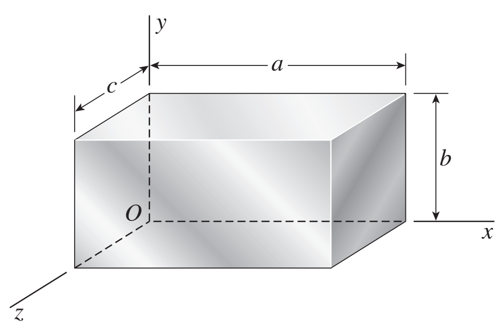
$U = uV_0 = 25.0\ \text{N} \cdot \text{m} = 25.0$ J ⟵

Triaxial Stress

When solving the problems for Section 7.6, assume that the material is linearly elastic with modulus of elasticity E and Poisson's ratio ν.

Problem 7.6-1 An element of aluminum in the form of a rectangular parallelepiped (see figure) of dimensions $a = 6.0$ in., $b = 4.0$ in, and $c = 3.0$ in. is subjected to *triaxial stresses* $\sigma_x = 12{,}000$ psi, $\sigma_y = -4{,}000$ psi, and $\sigma_z = -1{,}000$ psi acting on the x, y, and z faces, respectively.

Determine the following quantities: (a) the maximum shear stress τ_{max} in the material; (b) the changes Δa, Δb, and Δc in the dimensions of the element; (c) the change ΔV in the volume; and (d) the strain energy U stored in the element. (Assume $E = 10{,}400$ ksi and $\nu = 0.33$.)



Probs. 7.6-1 and 7.6-2

Solution 7.6-1 **Triaxial stress**

$\sigma_x = 12{,}000$ psi $\quad \sigma_y = -4{,}000$ psi
$\sigma_z = -1{,}000$ psi
$a = 6.0$ in. $\quad b = 4.0$ in. $\quad c = 3.0$ in.
$E = 10{,}400$ ksi $\quad \nu = 0.33$ (aluminum)

(a) MAXIMUM SHEAR STRESS

$\sigma_1 = 12{,}000$ psi $\quad \sigma_2 = -1{,}000$ psi
$\sigma_3 = -4{,}000$ psi

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = 8{,}000 \text{ psi} \quad \longleftarrow$$

(b) CHANGES IN DIMENSIONS

Eq. (7-53a): $\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z) = 1312.5 \times 10^{-6}$

Eq. (7-53b): $\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_z + \sigma_x) = -733.7 \times 10^{-6}$

Eq. (7-53c): $\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y) = -350.0 \times 10^{-6}$

$\Delta a = a\varepsilon_x = 0.0079$ in. (increase)
$\Delta b = b\varepsilon_y = -0.0029$ in. (decrease) $\longleftarrow$
$\Delta c = c\varepsilon_z = -0.0011$ in. (decrease)

(c) CHANGE IN VOLUME

Eq. (7-56):

$$e = \frac{1 - 2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = 228.8 \times 10^{-6}$$

$V = abc$
$\Delta V = e\,(abc) = 0.0165 \text{ in.}^3$ (increase) $\longleftarrow$

(d) STRAIN ENERGY

Eq. (7-57a): $u = \frac{1}{2}(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z)$
$= 9.517$ psi

$U = u\,(abc) = 685$ in.-lb $\longleftarrow$

Problem 7.6-2 Solve the preceding problem if the element is steel ($E = 200$ GPa, $\nu = 0.30$) with dimensions $a = 300$ mm, $b = 150$ mm, and $c = 150$ mm and the stresses are $\sigma_x = -60$ MPa, $\sigma_y = -40$ MPa, and $\sigma_z = -40$ MPa.

Solution 7.6-2 **Triaxial stress**

$\sigma_x = -60$ MPa $\quad \sigma_y = -40$ MPa
$\sigma_z = -40$ MPa
$a = 300$ mm $\quad b = 150$ mm $\quad c = 150$ mm
$E = 200$ GPa $\quad \nu = 0.30$ (steel)

(a) MAXIMUM SHEAR STRESS

$\sigma_1 = -40$ MPa $\quad \sigma_2 = -40$ MPa
$\sigma_3 = -60$ MPa

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = 10.0 \text{ MPa} \quad \longleftarrow$$

(b) CHANGES IN DIMENSIONS

Eq. (7-53a): $\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z) = -180.0 \times 10^{-6}$

Eq. (7-53b): $\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_z + \sigma_x) = -50.0 \times 10^{-6}$

Eq. (7-53c): $\varepsilon_z = \frac{\sigma_z}{E} - \frac{\nu}{E}(\sigma_x + \sigma_y) = -50.0 \times 10^{-6}$

$\Delta a = a\varepsilon_x = -0.0540$ mm (decrease)
$\Delta b = b\varepsilon_y = -0.0075$ mm (decrease) $\longleftarrow$
$\Delta c = c\varepsilon_z = -0.0075$ mm (decrease)

(c) CHANGE IN VOLUME

Eq. (7-56):

$$e = \frac{1 - 2\nu}{E}(\sigma_x + \sigma_y + \sigma_z) = -280.0 \times 10^{-6}$$

$V = abc$
$\Delta V = e\,(abc) = -1890 \text{ mm}^3$ (decrease) $\longleftarrow$

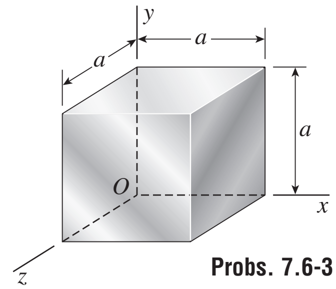
(d) STRAIN ENERGY

Eq. (7-57a): $u = \frac{1}{2}(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z)$
$= 0.00740$ MPa

$U = u\,(abc) = 50.0 \text{ N} \cdot \text{m} = 50.0 \text{ J}$ $\longleftarrow$

Problem 7.6-3 A cube of cast iron with sides of length $a = 4.0$ in. (see figure) is tested in a laboratory under *triaxial stress*. Gages mounted on the testing machine show that the compressive strains in the material are $\epsilon_x = -225 \times 10^{-6}$ and $\epsilon_y = \epsilon_z = -37.5 \times 10^{-6}$.

Determine the following quantities: (a) the normal stresses σ_x, σ_y, and σ_z acting on the x, y, and z faces of the cube; (b) the maximum shear stress τ_{max} in the material; (c) the change ΔV in the volume of the cube; and (d) the strain energy U stored in the cube. (Assume $E = 14{,}000$ ksi and $\nu = 0.25$.)



Probs. 7.6-3 and 7.6-4

Solution 7.6-3 Triaxial stress (cube)

$\varepsilon_x = -225 \times 10^{-6}$ $\varepsilon_y = -37.5 \times 10^{-6}$

$\varepsilon_z = -37.5 \times 10^{-6}$ $a = 4.0$ in.

$E = 14{,}000$ ksi $\nu = 0.25$ (cast iron)

(a) NORMAL STRESSES

Eq. (7-54a):

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_x + \nu(\varepsilon_y + \varepsilon_z)]$$

$= -4200$ psi ←

In a similar manner, Eqs. (7-54 b and c) give

$\sigma_y = -2100$ psi $\sigma_z = -2100$ psi ←

(b) MAXIMUM SHEAR STRESS

$\sigma_1 = -2100$ psi $\sigma_2 = -2100$ psi

$\sigma_3 = -4200$ psi

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = 1050 \text{ psi}$$ ←

(c) CHANGE IN VOLUME

Eq. (7-55): $e = \varepsilon_x + \varepsilon_y + \varepsilon_z = -0.000300$

$V = a^3$

$\Delta V = ea^3 = -0.0192$ in.^{3} (decrease) ←

(d) STRAIN ENERGY

Eq. (7-57a): $u = \frac{1}{2}(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z)$

$= 0.55125$ psi

$U = ua^3 = 35.3$ in.-lb ←

Problem 7.6-4 Solve the preceding problem if the cube is granite ($E = 60$ GPa, $\nu = 0.25$) with dimensions $a = 75$ mm and compressive strains $\epsilon_x = -720 \times 10^{-6}$ and $\epsilon_y = \epsilon_z = -270 \times 10^{-6}$.

Solution 7.6-4 Triaxial stress (cube)

$\varepsilon_x = -720 \times 10^{-6}$ $\varepsilon_y = -270 \times 10^{-6}$

$\varepsilon_z = -270 \times 10^{-6}$ $a = 75$ mm $E = 60$ GPa

$\nu = 0.25$ (Granite)

(a) NORMAL STRESSES

Eq. (7-54a):

$$\sigma_x = \frac{E}{(1+\nu)(1-2\nu)}[(1-\nu)\varepsilon_x + \nu(\varepsilon_x + \varepsilon_z)]$$

$= -64.8$ MPa ←

In a similar manner, Eqs. (7-54 b and c) give

$\sigma_y = -43.2$ MPa $\sigma_z = -43.2$ MPa ←

(b) MAXIMUM SHEAR STRESS

$\sigma_1 = -43.2$ MPa $\sigma_2 = -43.2$ MPa

$\sigma_3 = -64.8$ MPa

$$\tau_{max} = \frac{\sigma_1 - \sigma_3}{2} = 10.8 \text{ MPa}$$ ←

(c) CHANGE IN VOLUME

Eq. (7-55): $e = \varepsilon_x + \varepsilon_y + \varepsilon_z = -1260 \times 10^{-6}$

$V = a^3$

$\Delta V = ea^3 = -532$ mm^3 (decrease) ←

(d) STRAIN ENERGY

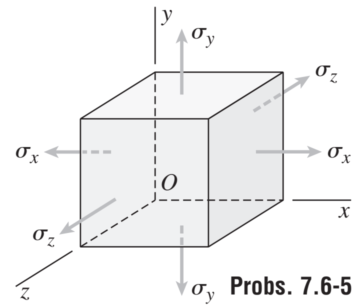
Eq. (7-57a): $u = \frac{1}{2}(\sigma_x\varepsilon_x + \sigma_y\varepsilon_y + \sigma_z\varepsilon_z)$

$= 0.03499$ MPa $= 34.99$ kPa

$U = ua^3 = 14.8$ N · m $= 14.8$ J ←

Problem 7.6-5 An element of aluminum in *triaxial stress* (see figure) is subjected to stresses $\sigma_x = 5200$ psi (tension), $\sigma_y = -4750$ psi (compression), and $\sigma_z = -3090$ psi (compression). It is also known that the normal strains in the x and y directions are $\epsilon_x = 713.8 \times 10^{-6}$ (elongation) and $\epsilon_y = -502.3 \times 10^{-6}$ (shortening).

What is the bulk modulus K for the aluminum?



Probs. 7.6-5 and 7.6-6

Solution 7.6-5 Triaxial stress (bulk modulus)

$\sigma_x = 5200$ psi $\quad \sigma_y = -4750$ psi

$\sigma_z = -3090$ psi $\quad \varepsilon_x = 713.8 \times 10^{-6}$

$\varepsilon_y = -502.3 \times 10^{-6}$

Find K.

Eq. (7-53a): $\varepsilon_x = \dfrac{\sigma_x}{E} - \dfrac{\nu}{E}(\sigma_y + \sigma_z)$

Eq. (7-53b): $\varepsilon_y = \dfrac{\sigma_y}{E} - \dfrac{\nu}{E}(\sigma_z + \sigma_x)$

Substitute numerical values and rearrange:

$(713.8 \times 10^{-6})\, E = 5200 + 7840\, \nu$ (1)

$(-502.3 \times 10^{-6})\, E = -4750 - 2110\, \nu$ (2)

Units: E = psi

Solve simultaneously Eqs. (1) and (2):

$E = 10.801 \times 10^6$ psi $\quad \nu = 0.3202$

Eq. (7-61): $K = \dfrac{E}{3(1 - 2\nu)} = 10.0 \times 10^6$ psi ⟵

Problem 7.6-6 Solve the preceding problem if the material is nylon subjected to compressive stresses $\sigma_x = -4.5$ MPa, $\sigma_y = -3.6$ MPa, and $\sigma_z = -2.1$ MPa, and the normal strains are $\epsilon_x = -740 \times 10^{-6}$ and $\epsilon_y = -320 \times 10^{-6}$ (shortenings).

Solution 7.6-6 Triaxial stress (bulk modulus)

$\sigma_x = -4.5$ MPa $\quad \sigma_y = -3.6$ MPa

$\sigma_z = -2.1$ MPa $\quad \varepsilon_x = -740 \times 10^{-6}$

$\varepsilon_y = -320 \times 10^{-6}$

Find K.

Eq. (7-53a): $\varepsilon_x = \dfrac{\sigma_x}{E} - \dfrac{\nu}{E}(\sigma_y + \sigma_z)$

Eq. (7-53b): $\varepsilon_y = \dfrac{\sigma_y}{E} - \dfrac{\nu}{E}(\sigma_z + \sigma_x)$

Substitute numerical values and rearrange:

$(-740 \times 10^{-6})\, E = -4.5 + 5.7\, \nu$ (1)

$(-320 \times 10^{-6})\, E = -3.6 + 6.6\, \nu$ (2)

Units: E = MPa

Solve simultaneously Eqs. (1) and (2):

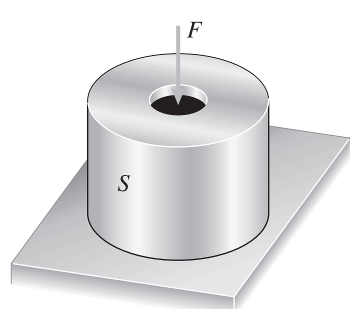
$E = 3{,}000$ MPa $= 3.0$ GPa $\quad \nu = 0.40$

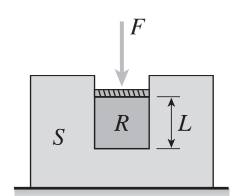
Eq. (7-61): $K = \dfrac{E}{3(1 - 2\nu)} = 5.0$ GPa ⟵

Problem 7.6-7 A rubber cylinder R of length L and cross-sectional area A is compressed inside a steel cylinder S by a force F that applies a uniformly distributed pressure to the rubber (see figure).

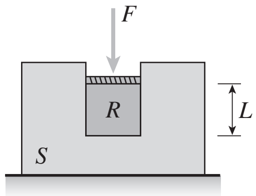
(a) Derive a formula for the lateral pressure p between the rubber and the steel. (Disregard friction between the rubber and the steel, and assume that the steel cylinder is rigid when compared to the rubber.)

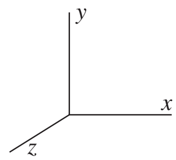
(b) Derive a formula for the shortening δ of the rubber cylinder.

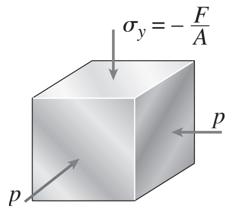




Solution 7.6-7 Rubber cylinder







$$\sigma_x = -p \qquad \sigma_y = -\frac{F}{A}$$
$$\sigma_z = -p$$
$$\varepsilon_x = \varepsilon_z = 0$$

(a) Lateral pressure

Eq. (7-53a): $\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$

OR $0 = -p - \nu\left(-\frac{F}{A} - p\right)$

Solve for p: $p = \frac{\nu}{1-\nu}\left(\frac{F}{A}\right)$ ←

(b) Shortening

Eq. (7-53b): $\varepsilon_y = \frac{\sigma_y}{E} - \frac{\nu}{E}(\sigma_z + \sigma_x)$

$$= -\frac{F}{EA} - \frac{\nu}{E}(-2p)$$

Substitute for p and simplify:

$$\varepsilon_y = \frac{F}{EA}\,\frac{(1+\nu)(-1+2\nu)}{1-\nu}$$

(Positive ε_y represents an increase in strain, that is, elongation.)

$\delta = -\varepsilon_y L$

$$\delta = \frac{(1+\nu)(1-2\nu)}{(1-\nu)}\left(\frac{FL}{EA}\right) \quad \leftarrow$$

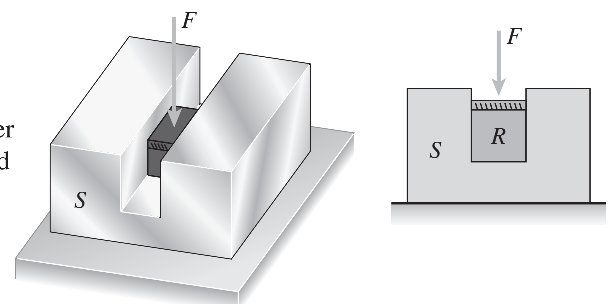
(Positive δ represents a shortening of the rubber cylinder.)

Problem 7.6-8 A block R of rubber is confined between plane parallel walls of a steel block S (see figure). A uniformly distributed pressure p_0 is applied to the top of the rubber block by a force F.

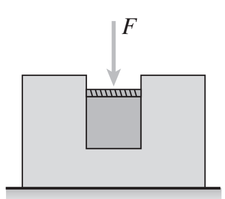
(a) Derive a formula for the lateral pressure p between the rubber and the steel. (Disregard friction between the rubber and the steel, and assume that the steel block is rigid when compared to the rubber.)

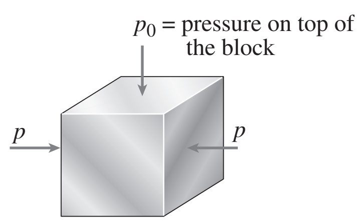
(b) Derive a formula for the dilatation e of the rubber.

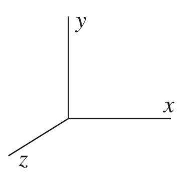
(c) Derive a formula for the strain-energy density u of the rubber.



Solution 7.6-8 Block of rubber







$$\sigma_x = -p$$
$$\sigma_y = -p_0 \qquad \sigma_z = 0$$
$$\varepsilon_x = 0 \qquad \varepsilon_y \neq 0 \qquad \varepsilon_z \neq 0$$

(a) Lateral pressure

Eq. (7-53a): $\varepsilon_x = \frac{\sigma_x}{E} - \frac{\nu}{E}(\sigma_y + \sigma_z)$

OR $0 = -p - \nu(-p_0) \qquad \therefore p = \nu p_0$ ←

(b) Dilatation

Eq. (7-56): $e = \frac{1-2\nu}{E}(\sigma_x + \sigma_y + \sigma_z)$

$$= \frac{1-2\nu}{E}(-p - p_0)$$

Substitute for p:

$$e = -\frac{(1+\nu)(1-2\nu)p_0}{E} \quad \leftarrow$$

(c) Strain energy density

Eq. (7-57b):

$$u = \frac{1}{2E}(\sigma_x^2 + \sigma_y^2 + \sigma_z^2) - \frac{\nu}{E}(\sigma_x\sigma_y + \sigma_x\sigma_z + \sigma_y\sigma_z)$$

Substitute for σ_x, σ_y, σ_z, and p:

$$u = \frac{(1-\nu^2)p_0^2}{2E} \quad \leftarrow$$

Problem 7.6-9 A solid spherical ball of brass ($E = 15 \times 10^6$ psi, $\nu = 0.34$) is lowered into the ocean to a depth of 10,000 ft. The diameter of the ball is 11.0 in.

Determine the decrease Δd in diameter, the decrease ΔV in volume, and the strain energy U of the ball.

Solution 7.6-9 Brass sphere

$E = 15 \times 10^6$ psi $\quad \nu = 0.34$

Lowered in the ocean to depth $h = 10{,}000$ ft

Diameter $d = 11.0$ in.

Sea water: $\gamma = 63.8$ lb/ft^3

Pressure: $\sigma_0 = \gamma h = 638{,}000$ lb/ft^2 $= 4431$ psi

DECREASE IN DIAMETER

$$\text{Eq. (7-59): } \varepsilon_0 = \frac{\sigma_0}{E}(1 - 2\nu) = 94.53 \times 10^{-6}$$

$$\Delta d = \varepsilon_0 d = 1.04 \times 10^{-3} \text{ in.} \quad \longleftarrow$$

(decrease)

DECREASE IN VOLUME

Eq. (7-60): $e = 3\varepsilon_0 = 283.6 \times 10^{-6}$

$$V_0 = \frac{4}{3}\pi r^3 = \frac{4}{3}(\pi)\left(\frac{11.0 \text{ in.}}{2}\right)^3 = 696.9 \text{ in.}^3$$

$$\Delta V = eV_0 = 0.198 \text{ in.}^3 \quad \longleftarrow$$

(decrease)

STRAIN ENERGY

Use Eq. (7-57b) with $\sigma_x = \sigma_y = \sigma_z = \sigma_0$:

$$u = \frac{3(1 - 2\nu)\sigma_0^2}{2E} = 0.6283 \text{ psi}$$

$$U = uV_0 = 438 \text{ in.-lb} \quad \longleftarrow$$

Problem 7.6-10 A solid steel sphere ($E = 210$ GPa, $\nu = 0.3$) is subjected to hydrostatic pressure p such that its volume is reduced by 0.4%.

(a) Calculate the pressure p.

(b) Calculate the volume modulus of elasticity K for the steel.

(c) Calculate the strain energy U stored in the sphere if its diameter is $d = 150$ mm.

Solution 7.6-10 Steel sphere

$E = 210$ GPa $\quad \nu = 0.3$

Hydrostatic Pressure. V_0 = Initial volume

$\Delta V = 0.004V_0$

$$\text{Dilatation: } e = \frac{\Delta V}{V_0} = 0.004$$

(a) PRESSURE

$$\text{Eq. (7-60): } e = \frac{3\sigma_0(1 - 2\nu)}{E}$$

$$\text{or} \quad \sigma_0 = \frac{Ee}{3(1 - 2\nu)} = 700 \text{ MPa}$$

Pressure $p = \sigma_0 = 700$ MPa $\longleftarrow$

(b) VOLUME MODULUS OF ELASTICITY

$$\text{Eq. (7-63): } K = \frac{\sigma_0}{E} = \frac{700 \text{ MPa}}{0.004} = 175 \text{ GPa} \quad \longleftarrow$$

(c) STRAIN ENERGY (d = diameter)

$d = 150$ mm $\quad r = 75$ mm

From Eq. (7-57b) with $\sigma_x = \sigma_y = \sigma_z = \sigma_0$:

$$u = \frac{3(1 - 2\nu)\sigma_0^2}{2E} = 1.40 \text{ MPa}$$

$$V_0 = \frac{4\pi r^3}{3} = 1767 \times 10^{-6} \text{ m}^3$$

$$U = uV_0 = 2470 \text{ N} \cdot \text{m} = 2470 \text{ J} \quad \longleftarrow$$

Problem 7.6-11 A solid bronze sphere (volume modulus of elasticity $K = 14.5 \times 10^6$ psi) is suddenly heated around its outer surface. The tendency of the heated part of the sphere to expand produces uniform tension in all directions at the center of the sphere.

If the stress at the center is 12,000 psi, what is the strain? Also, calculate the unit volume change e and the strain-energy density u at the center.

Solution 7.6-11 Bronze sphere (heated)

$K = 14.5 \times 10^6$ psi

$\sigma_0 = 12{,}000$ psi (tension at the center)

STRAIN AT THE CENTER OF THE SPHERE

Eq. (7-59): $\varepsilon_0 = \dfrac{\sigma_0}{E}(1 - 2\nu)$

Eq. (7-61): $K = \dfrac{E}{3(1 - 2\nu)}$

Combine the two equations:

$\varepsilon_0 = \dfrac{\sigma_0}{3K} = 276 \times 10^{-6}$ ←

UNIT VOLUME CHANGE AT THE CENTER

Eq. (7-62): $e = \dfrac{\sigma_0}{K} = 828 \times 10^{-6}$ ←

STRAIN ENERGY DENSITY AT THE CENTER

Eq. (7-57b) with $\sigma_x = \sigma_y = \sigma_z = \sigma_0$:

$u = \dfrac{3(1 - 2\nu)\sigma_0^2}{2E} = \dfrac{\sigma_0^2}{2K}$

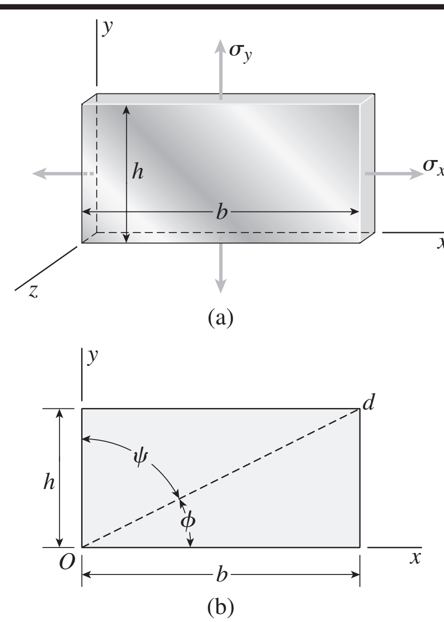
$u = 4.97$ psi ←

Plane Strain

When solving the problems for Section 7.7, consider only the in-plane strains (the strains in the xy plane) unless stated otherwise. Use the transformation equations of plane strain except when Mohr's circle is specified (Problems 7.7-23 through 7.7-28).

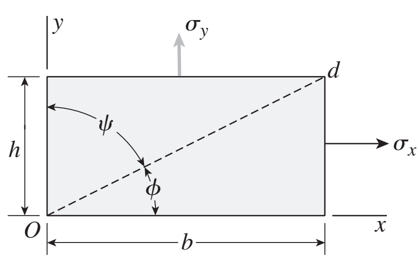
Problem 7.7-1 A thin rectangular plate in *biaxial stress* is subjected to stresses σ_x and σ_y, as shown in part (a) of the figure on the next page. The width and height of the plate are $b = 8.0$ in. and $h = 4.0$ in., respectively. Measurements show that the normal strains in the x and y directions are $\epsilon_x = 195 \times 10^{-6}$ and $\epsilon_y = -125 \times 10^{-6}$, respectively.

With reference to part (b) of the figure, which shows a two-dimensional view of the plate, determine the following quantities: (a) the increase Δd in the length of diagonal Od; (b) the change $\Delta\phi$ in the angle ϕ between diagonal Od and the x axis; and (c) the change $\Delta\psi$ in the angle ψ between diagonal Od and the y axis.



Probs. 7.7-1 and 7.7-2

Solution 7.7-1 Plate in biaxial stress



$b = 8.0$ in. $\quad h = 4.0$ in. $\quad \varepsilon_x = 195 \times 10^{-6}$

$\varepsilon_y = -125 \times 10^{-6} \quad \gamma_{xy} = 0$

$\phi = \arctan \dfrac{h}{b} = 26.57°$

$L_d = \sqrt{b^2 + h^2} = 8.944$ in.

(a) INCREASE IN LENGTH OF DIAGONAL

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

For $\theta = \phi = 26.57°$, $\varepsilon_{x_1} = 130.98 \times 10^{-6}$

$\Delta d = \varepsilon_{x_1} L_d = 0.00117$ in. ←

(b) CHANGE IN ANGLE ϕ

Eq. (7-68): $\alpha = -(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta - \gamma_{xy}\sin^2\theta$
For $\theta = \phi = 26.57°$: $\alpha = -128.0 \times 10^{-6}$ rad
Minus sign means line Od rotates clockwise (angle ϕ decreases).
$\Delta\phi = 128 \times 10^{-6}$ rad (decrease) ⟵

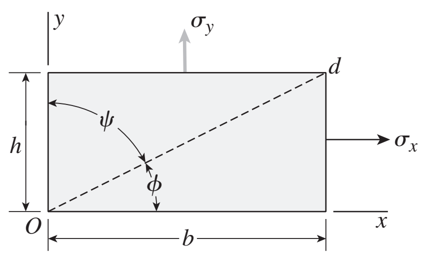
(c) CHANGE IN ANGLE ψ

Angle ψ increases the same amount that ϕ decreases.

$\Delta\psi = 128 \times 10^{-6}$ rad (increase) ⟵

Problem 7.7-2 Solve the preceding problem if $b = 160$ mm, $h = 60$ mm, $\epsilon_x = 410 \times 10^{-6}$, and $\epsilon_y = -320 \times 10^{-6}$.

Solution 7.7-2 Plate in biaxial stress



$b = 160$ mm $\quad h = 60$ mm $\quad \varepsilon_x = 410 \times 10^{-6}$
$\varepsilon_y = -320 \times 10^{-6} \quad \gamma_{xy} = 0$

$$\phi = \arctan\frac{h}{b} = 20.56°$$

$$L_d = \sqrt{b^2 + h^2} = 170.88 \text{ mm}$$

(a) INCREASE IN LENGTH OF DIAGONAL

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

For $\theta = \phi = 20.56°$: $\varepsilon_{x_1} = 319.97 \times 10^{-6}$
$\Delta d = \varepsilon_{x_1} L_d = 0.0547$ mm ⟵

(b) CHANGE IN ANGLE ϕ

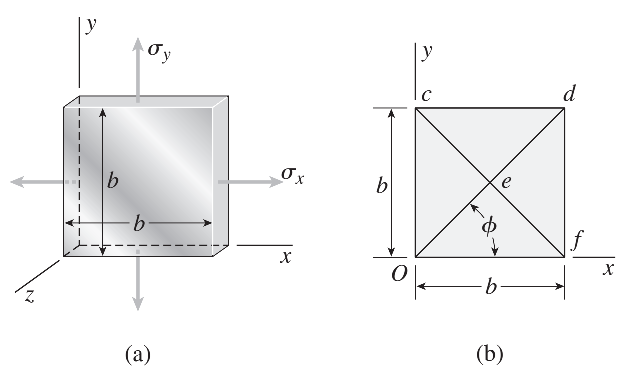
Eq. (7-68): $\alpha = -(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta - \gamma_{xy}\sin^2\theta$
For $\theta = \phi = 20.56°$: $\alpha = -240.0 \times 10^{-6}$ rad
Minus sign means line Od rotates clockwise (angle ϕ decreases).
$\Delta\phi = 240 \times 10^{-6}$ rad (decrease) ⟵

(c) CHANGE IN ANGLE ψ

Angle ψ increases the same amount that ϕ decreases.
$\Delta\psi = 240 \times 10^{-6}$ rad (increase) ⟵

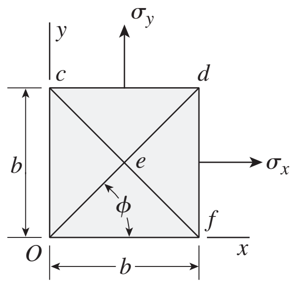
Problem 7.7-3 A thin square plate in *biaxial stress* is subjected to stresses σ_x and σ_y, as shown in part (a) of the figure. The width of the plate is $b = 12.0$ in. Measurements show that the normal strains in the x and y directions are $\epsilon_x = 427 \times 10^{-6}$ and $\epsilon_y = 113 \times 10^{-6}$, respectively.

With reference to part (b) of the figure, which shows a two-dimensional view of the plate, determine the following quantities: (a) the increase Δd in the length of diagonal Od; (b) the change $\Delta\phi$ in the angle ϕ between diagonal Od and the x axis; and (c) the shear strain γ associated with diagonals Od and cf (that is, find the decrease in angle ced).



Probs. 7.7-3 and 7.7-4

Solution 7.7-3 Square plate in biaxial stress



$b = 12.0$ in. $\quad \varepsilon_x = 427 \times 10^{-6}$

$\varepsilon_y = 113 \times 10^{-6}$

$\phi = 45° \quad \gamma_{xy} = 0$

$L_d = b\sqrt{2} = 16.97$ in.

(a) INCREASE IN LENGTH OF DIAGONAL

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

For $\theta = \phi = 45°$: $\varepsilon_{x_1} = 270 \times 10^{-6}$

$\Delta d = \varepsilon_{x_1} L_d = 0.00458$ in. ⟵

(b) CHANGE IN ANGLE ϕ

Eq. (7-68): $\alpha = -(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta - \gamma_{xy}\sin^2\theta$

For $\theta = \phi = 45°$: $\alpha = -157 \times 10^{-6}$ rad

Minus sign means line *Od* rotates clockwise (angle ϕ decreases).

$\Delta\phi = 157 \times 10^{-6}$ rad (decrease) ⟵

(c) SHEAR STRAIN BETWEEN DIAGONALS

Eq. (7-71b): $$\frac{\gamma_{x_1y_1}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

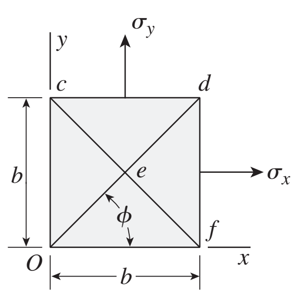
For $\theta = \phi = 45°$: $\gamma_{x_1y_1} = -314 \times 10^{-6}$ rad

(Negative strain means angle *ced* increases)

$\gamma = -314 \times 10^{-6}$ rad ⟵

Problem 7.7-4 Solve the preceding problem if $b = 225$ mm, $\epsilon_x = 845 \times 10^{-6}$, and $\epsilon_y = 211 \times 10^{-6}$.

Solution 7.7-4 Square plate in biaxial stress



$b = 225$ mm $\quad \varepsilon_x = 845 \times 10^{-6}$

$\varepsilon_y = 211 \times 10^{-6} \quad \phi = 45° \quad \gamma_{xy} = 0$

$L_d = b\sqrt{2} = 318.2$ mm

(a) INCREASE IN LENGTH OF DIAGONAL

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

For $\theta = \phi = 45°$: $\varepsilon_{x_1} = 528 \times 10^{-6}$

$\Delta d = \varepsilon_{x_1} L_d = 0.168$ mm ⟵

(b) CHANGE IN ANGLE ϕ

Eq. (7-68): $\alpha = -(\varepsilon_x - \varepsilon_y)\sin\theta\cos\theta - \gamma_{xy}\sin^2\theta$

For $\theta = \phi = 45°$: $\alpha = -317 \times 10^{-6}$ rad

Minus sign means line *Od* rotates clockwise (angle ϕ decreases).

$\Delta\phi = 317 \times 10^{-6}$ rad (decrease) ⟵

(c) SHEAR STRAIN BETWEEN DIAGONALS

Eq. (7-71b): $$\frac{\gamma_{x_1y_1}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

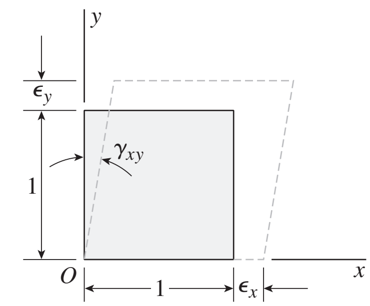
For $\theta = \phi = 45°$: $\gamma_{x_1y_1} = -634 \times 10^{-6}$ rad

(Negative strain means angle *ced* increases)

$\gamma = -634 \times 10^{-6}$ rad ⟵

Problem 7.7-5 An element of material subjected to *plane strain* (see figure) has strains as follows: $\epsilon_x = 220 \times 10^{-6}$, $\epsilon_y = 480 \times 10^{-6}$, and $\gamma_{xy} = 180 \times 10^{-6}$.

Calculate the strains for an element oriented at an angle $\theta = 50°$ and show these strains on a sketch of a properly oriented element.



Probs. 7.7-5 through 7.7-10

Solution 7.7-5 Element in plane strain

$\varepsilon_x = 220 \times 10^{-6} \qquad \varepsilon_y = 480 \times 10^{-6}$

$\gamma_{xy} = 180 \times 10^{-6}$

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

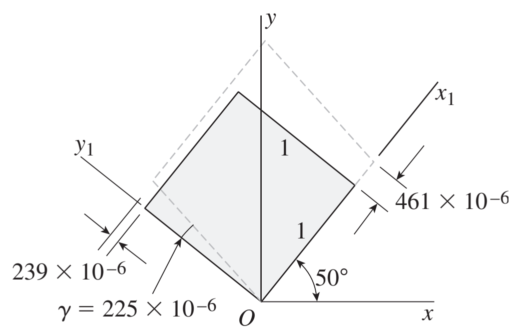
$$\frac{\gamma_{x_1y_1}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\varepsilon_{y_1} = \varepsilon_x + \varepsilon_y - \varepsilon_{x_1}$$

FOR $\theta = 50°$:

$\varepsilon_{x_1} = 461 \times 10^{-6} \qquad \gamma_{x_1y_1} = 225 \times 10^{-6}$

$\varepsilon_{y_1} = 239 \times 10^{-6}$



Problem 7.7-6 Solve the preceding problem for the following data: $\epsilon_x = 420 \times 10^{-6}$, $\epsilon_y = -170 \times 10^{-6}$, $\gamma_{xy} = 310 \times 10^{-6}$, and $\theta = 37.5°$.

Solution 7.7-6 Element in plane strain

$\varepsilon_x = 420 \times 10^{-6} \qquad \varepsilon_y = -170 \times 10^{-6}$

$\gamma_{xy} = 310 \times 10^{-6}$

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

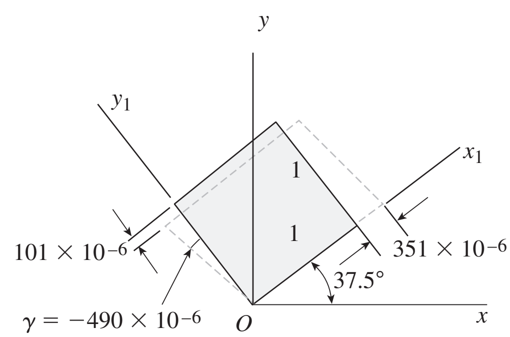
$$\frac{\gamma_{x_1y_1}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$\varepsilon_{y_1} = \varepsilon_x + \varepsilon_y - \varepsilon_{x_1}$$

FOR $\theta = 37.5°$:

$\varepsilon_{x_1} = 351 \times 10^{-6} \qquad \gamma_{x_1y_1} = -490 \times 10^{-6}$

$\varepsilon_{y_1} = -101 \times 10^{-6}$



Problem 7.7-7 The strains for an element of material in *plane strain* (see figure) are as follows: $\epsilon_x = 480 \times 10^{-6}$, $\epsilon_y = 140 \times 10^{-6}$, and $\gamma_{xy} = -350 \times 10^{-6}$.

Determine the principal strains and maximum shear strains, and show these strains on sketches of properly oriented elements.

Solution 7.7-7 Element in plane strain

$\varepsilon_x = 480 \times 10^{-6}$ $\quad\varepsilon_y = 140 \times 10^{-6}$

$\gamma_{xy} = -350 \times 10^{-6}$

Principal strains

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$= 310 \times 10^{-6} \pm 244 \times 10^{-6}$

$\varepsilon_1 = 554 \times 10^{-6} \quad \varepsilon_2 = 66 \times 10^{-6}$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = -1.0294$$

$2\theta_p = -45.8^\circ$ and 134.2°

$\theta_p = -22.9^\circ$ and 67.1°

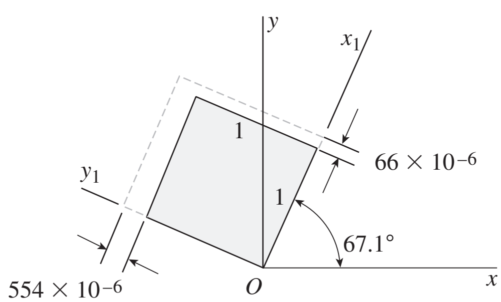
For $\theta_p = -22.9^\circ$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$= 554 \times 10^{-6}$

$\therefore\ \theta_{p_1} = -22.9^\circ \quad \varepsilon_1 = 554 \times 10^{-6}$ ⟵

$\theta_{p_2} = 67.1^\circ \quad \varepsilon_2 = 66 \times 10^{-6}$ ⟵



Maximum shear strains

$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$= 244 \times 10^{-6}$

$\gamma_{max} = 488 \times 10^{-6}$

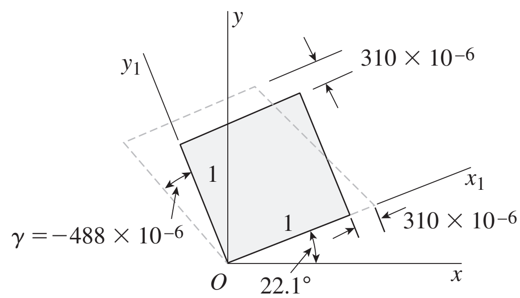
$\theta_{s_1} = \theta_{p_1} - 45^\circ = -67.9^\circ$ or 112.1°

$\gamma_{max} = 488 \times 10^{-6}$ ⟵

$\theta_{s_2} = \theta_{s_1} + 90^\circ = 22.1^\circ$

$\gamma_{min} = -488 \times 10^{-6}$ ⟵

$$\varepsilon_{aver} = \frac{\varepsilon_x + \varepsilon_y}{2} = 310 \times 10^{-6}$$



Problem 7.7-8 Solve the preceding problem for the following strains: $\epsilon_x = 120 \times 10^{-6}$, $\epsilon_y = -450 \times 10^{-6}$, and $\gamma_{xy} = -360 \times 10^{-6}$.

Solution 7.7-8 Element in plane strain

$\varepsilon_x = 120 \times 10^{-6} \quad \varepsilon_y = -450 \times 10^{-6}$

$\gamma_{xy} = -360 \times 10^{-6}$

Principal strains

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$= -165 \times 10^{-6} \pm 377 \times 10^{-6}$

$\varepsilon_1 = 172 \times 10^{-6} \quad \varepsilon_2 = -502 \times 10^{-6}$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = -0.6316$$

$2\theta_p = 327.7^\circ$ and 147.7°

$\theta_p = 163.9^\circ$ and 73.9°

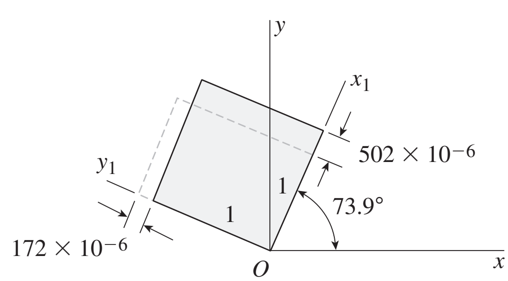
For $\theta_p = 163.9^\circ$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$= 172 \times 10^{-6}$

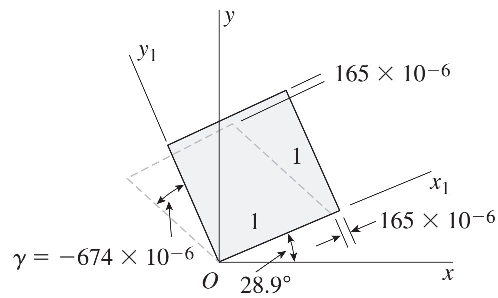
$\therefore\ \theta_{p_1} = 163.9^\circ \quad \varepsilon_1 = 172 \times 10^{-6}$ ⟵

$\theta_{p_2} = 73.9^\circ \quad \varepsilon_2 = -502 \times 10^{-6}$ ⟵



MAXIMUM SHEAR STRAINS

$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$
$$= 337 \times 10^{-6}$$
$$\gamma_{max} = 674 \times 10^{-6}$$
$$\theta_{s_1} = \theta_{p_1} - 45° = 118.9°$$
$$\gamma_{max} = 674 \times 10^{-6} \longleftarrow$$
$$\theta_{s_2} = \theta_{s_1} - 90° = 28.9°$$
$$\gamma_{min} = -674 \times 10^{-6} \longleftarrow$$
$$\varepsilon_{aver} = \frac{\varepsilon_x + \varepsilon_y}{2} = -165 \times 10^{-6}$$



Problem 7.7-9 An element of material in *plane strain* (see figure) is subjected to strains $\epsilon_x = 480 \times 10^{-6}$, $\epsilon_y = 70 \times 10^{-6}$, and $\gamma_{xy} = 420 \times 10^{-6}$.

Determine the following quantities: (a) the strains for an element oriented at an angle $\theta = 75°$, (b) the principal strains, and (c) the maximum shear strains. Show the results on sketches of properly oriented elements.

Solution 7.7-9 **Element in plane strain**

$\varepsilon_x = 480 \times 10^{-6}$ $\quad \varepsilon_y = 70 \times 10^{-6}$

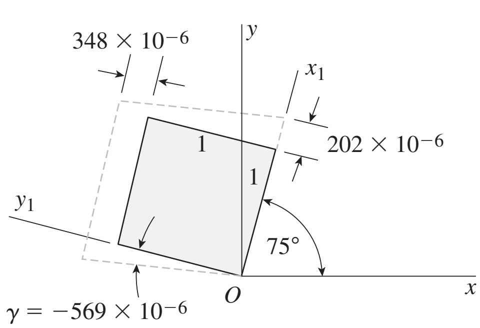
$\gamma_{xy} = 420 \times 10^{-6}$

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$\frac{\gamma_{x_1y_1}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$
$$\varepsilon_{y_1} = \varepsilon_x + \varepsilon_y - \varepsilon_{x_1}$$

FOR $\theta = 75°$:

$\varepsilon_{x_1} = 202 \times 10^{-6}$ $\quad \gamma_{x_1y_1} = -569 \times 10^{-6}$

$\varepsilon_{y_1} = 348 \times 10^{-6}$



PRINCIPAL STRAINS

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$
$$= 275 \times 10^{-6} \pm 293 \times 10^{-6}$$

$\varepsilon_1 = 568 \times 10^{-6}$ $\quad \varepsilon_2 = -18 \times 10^{-6}$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = 1.0244$$

$2\theta_p = 45.69°$ and $225.69°$

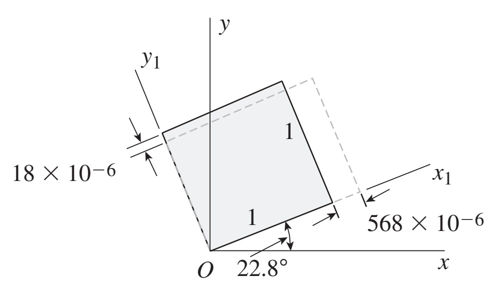
$\theta_p = 22.85°$ and $112.85°$

For $\theta_p = 22.85°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$
$$= 568 \times 10^{-6}$$

$\therefore \; \theta_{p_1} = 22.8°$ $\quad \varepsilon_1 = 568 \times 10^{-6} \longleftarrow$

$\theta_{p_2} = 112.8°$ $\quad \varepsilon_2 = -18 \times 10^{-6} \longleftarrow$



MAXIMUM SHEAR STRAINS

$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2} = 293 \times 10^{-6}$$

$\gamma_{max} = 587 \times 10^{-6}$

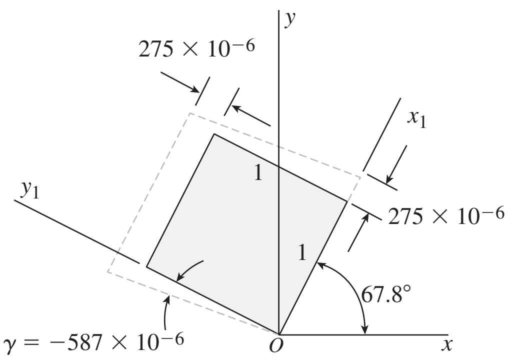
$\theta_{s_1} = \theta_{p_1} - 45° = -22.2°$ or $157.8°$

$\gamma_{max} = 587 \times 10^{-6}$ ←

$\theta_{s_2} = \theta_{s_1} + 90° = 67.8°$

$\gamma_{min} = -587 \times 10^{-6}$ ←

$$\varepsilon_{aver} = \frac{\varepsilon_x + \varepsilon_y}{2} = 275 \times 10^{-6}$$



Problem 7.7-10 Solve the preceding problem for the following data: $\epsilon_x = -1120 \times 10^{-6}$, $\epsilon_y = -430 \times 10^{-6}$, $\gamma_{xy} = 780 \times 10^{-6}$, and $\theta = 45°$.

Solution 7.7-10 Element in plane strain

$\varepsilon_x = -1120 \times 10^{-6}$ $\quad \varepsilon_y = -430 \times 10^{-6}$

$\gamma_{xy} = 780 \times 10^{-6}$

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

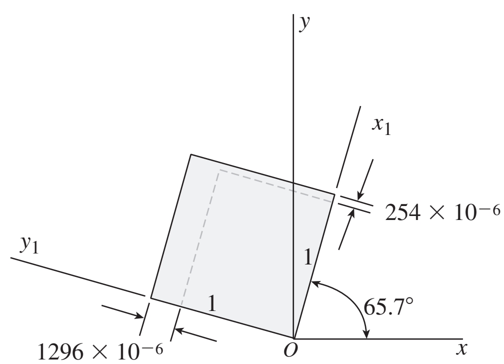
$$\frac{\gamma_{x_1y_1}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

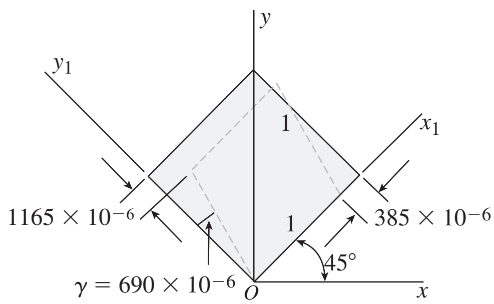
$\varepsilon_{y_1} = \varepsilon_x + \varepsilon_y - \varepsilon_{x_1}$

FOR $\theta = 45°$:

$\varepsilon_{x_1} = -385 \times 10^{-6}$ $\quad \gamma_{x_1y_1} = 690 \times 10^{-6}$

$\varepsilon_{y_1} = -1165 \times 10^{-6}$





PRINCIPAL STRAINS

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$= -775 \times 10^{-6} \pm 521 \times 10^{-6}$

$\varepsilon_1 = -254 \times 10^{-6}$ $\quad \varepsilon_2 = -1296 \times 10^{-6}$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = -1.1304$$

$2\theta_p = 131.5°$ and $311.5°$

$\theta_p = 65.7°$ and $155.7°$

For $\theta_p = 65.7°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$= -254 \times 10^{-6}$

$\therefore \theta_{p_1} = 65.7°$ $\quad \varepsilon_1 = -254 \times 10^{-6}$ ←

$\theta_{p_2} = 155.7°$ $\quad \varepsilon_2 = -1296 \times 10^{-6}$ ←

MAXIMUM SHEAR STRAINS

$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$= 521 \times 10^{-6}$

$\gamma_{max} = 1041 \times 10^{-6}$

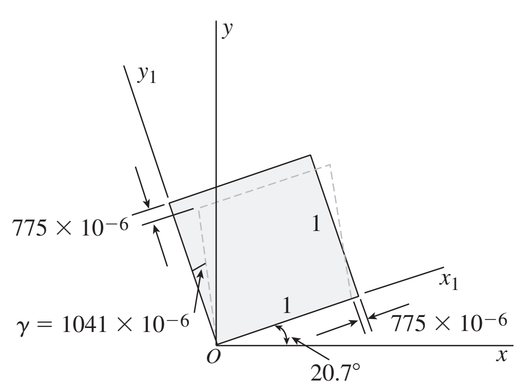
$\theta_{s_1} = \theta_{p_1} - 45° = 20.7°$

$\gamma_{max} = 1041 \times 10^{-6}$ ←

$\theta_{s_2} = \theta_{s_1} + 90° = 110.7°$

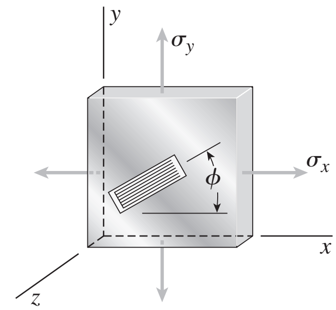
$\gamma_{min} = -1041 \times 10^{-6}$ ←

$$\varepsilon_{aver} = \frac{\varepsilon_x + \varepsilon_y}{2} = -775 \times 10^{-6}$$



Problem 7.7-11 A steel plate with modulus of elasticity $E = 30 \times 10^6$ psi and Poisson's ratio $\nu = 0.30$ is loaded in *biaxial stress* by normal stresses σ_x and σ_y (see figure). A strain gage is bonded to the plate at an angle $\phi = 30°$.

If the stress σ_x is 18,000 psi and the strain measured by the gage is $\epsilon = 407 \times 10^{-6}$, what is the maximum in-plane shear stress $(\tau_{max})_{xy}$ and shear strain $(\gamma_{max})_{xy}$? What is the maximum shear strain $(\gamma_{max})_{xz}$ in the xz plane? What is the maximum shear strain $(\gamma_{max})_{yz}$ in the yz plane?



Probs. 7.7-11 and 7.7-12

Solution 7.7-11 Steel plate in biaxial stress

$\sigma_x = 18{,}000$ psi $\quad \gamma_{xy} = 0 \quad \sigma_y = ?$
$E = 30 \times 10^6$ psi $\quad \nu = 0.30$
Strain gage: $\phi = 30° \quad \varepsilon = 407 \times 10^{-6}$

UNITS: All stresses in psi.

STRAIN IN BIAXIAL STRESS (EQS. 7-39)

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{30 \times 10^6}(18{,}000 - 0.3\sigma_y) \quad (1)$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{30 \times 10^6}(\sigma_y - 5400) \quad (2)$$

$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{0.3}{30 \times 10^6}(18{,}000 + \sigma_y) \quad (3)$$

STRAINS AT ANGLE $\phi = 30°$ (Eq. 7-71a)

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$407 \times 10^{-6} = \left(\frac{1}{2}\right)\left(\frac{1}{30 \times 10^6}\right)(12{,}600 + 0.7\sigma_y) + \left(\frac{1}{2}\right)\left(\frac{1}{30 \times 10^6}\right)(23{,}400 - 1.3\sigma_y)\cos 60°$$

Solve for σ_y: $\quad \sigma_y = 2400$ psi $\quad (4)$

MAXIMUM IN-PLANE SHEAR STRESS

$$(\tau_{max})_{xy} = \frac{\sigma_x - \sigma_y}{2} = 7800 \text{ psi} \quad \longleftarrow$$

STRAINS FROM EQS. (1), (2), AND (3)

$\varepsilon_x = 576 \times 10^{-6} \quad \varepsilon_y = -100 \times 10^{-6}$
$\varepsilon_z = -204 \times 10^{-6}$

MAXIMUM SHEAR STRAINS (EQ. 7-75)

xy plane: $$\frac{(\gamma_{max})_{xy}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$\gamma_{xy} = 0 \quad (\gamma_{max})_{xy} = 676 \times 10^{-6} \quad \longleftarrow$

xz plane: $$\frac{(\gamma_{max})_{xz}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_z}{2}\right)^2 + \left(\frac{\gamma_{xz}}{2}\right)^2}$$

$\gamma_{xz} = 0 \quad (\gamma_{max})_{xz} = 780 \times 10^{-6} \quad \longleftarrow$

yz plane: $$\frac{(\gamma_{max})_{yz}}{2} = \sqrt{\left(\frac{\varepsilon_y - \varepsilon_z}{2}\right)^2 + \left(\frac{\gamma_{yz}}{2}\right)^2}$$

$\gamma_{yz} = 0 \quad (\gamma_{max})_{yz} = 104 \times 10^{-6} \quad \longleftarrow$

Problem 7.7-12 Solve the preceding problem if the plate is made of aluminum with $E = 72$ GPa and $\nu = 1/3$, the stress σ_x is 86.4 MPa, the angle ϕ is 21°, and the strain ϵ is 946×10^{-6}.

Solution 7.7-12 Aluminum plate in biaxial stress

$\sigma_x = 86.4$ MPa $\quad \gamma_{xy} = 0 \quad \sigma_y = ?$
$E = 72$ GPa $\quad \nu = 1/3$
Strain gage: $\phi = 21° \quad \varepsilon = 946 \times 10^{-6}$

UNITS: All stresses in MPa.

STRAINS IN BIAXIAL STRESS (EQS. 7-39)

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = \frac{1}{72{,}000}\left(86.4 - \frac{1}{3}\sigma_y\right) \quad (1)$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = \frac{1}{72{,}000}(\sigma_y - 28.8) \quad (2)$$

$$\varepsilon_z = -\frac{\nu}{E}(\sigma_x + \sigma_y) = -\frac{1/3}{72{,}000}(86.4 + \sigma_y) \quad (3)$$

STRAINS AT ANGLE $\phi = 21°$ (EQ. 7-71a)

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$946 \times 10^{-6} = \left(\frac{1}{2}\right)\left(\frac{1}{72{,}000}\right)\left(57.6 + \frac{2}{3}\sigma_y\right) + \left(\frac{1}{2}\right)\left(\frac{1}{72{,}000}\right)\left(115.2 - \frac{4}{3}\sigma_y\right)\cos 42°$$

Solve for σ_y: $\sigma_y = 21.55$ MPa (4)

MAXIMUM IN-PLANE SHEAR STRESS

$$(\tau_{max})_{xy} = \frac{\sigma_x - \sigma_y}{2} = 32.4 \text{ MPa} \longleftarrow$$

STRAINS FROM EQS. (1), (2), AND (3)

$\varepsilon_x = 1100 \times 10^{-6}$ $\varepsilon_y = -101 \times 10^{-6}$
$\varepsilon_z = -500 \times 10^{-6}$

MAXIMUM SHEAR STRAINS (EQ. 7-75)

xy plane: $$\frac{(\gamma_{max})_{xy}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$\gamma_{xy} = 0$ $(\gamma_{max})_{xy} = 1200 \times 10^{-6}$ ⟵

xz plane: $$\frac{(\gamma_{max})_{xz}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_z}{2}\right)^2 + \left(\frac{\gamma_{xz}}{2}\right)^2}$$

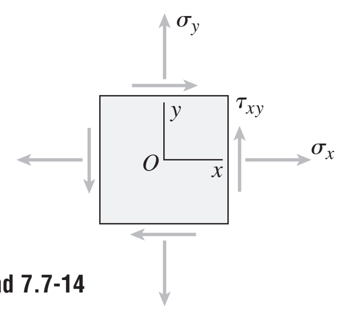
$\gamma_{xz} = 0$ $(\gamma_{max})_{xz} = 1600 \times 10^{-6}$ ⟵

yz plane: $$\frac{(\gamma_{max})_{yz}}{2} = \sqrt{\left(\frac{\varepsilon_y - \varepsilon_z}{2}\right)^2 + \left(\frac{\gamma_{yz}}{2}\right)^2}$$

$\gamma_{yz} = 0$ $(\gamma_{max})_{yz} = 399 \times 10^{-6}$ ⟵

Problem 7.7-13 An element in *plane stress* is subjected to stresses $\sigma_x = -8400$ psi, $\sigma_y = 1100$ psi, and $\tau_{xy} = -1700$ psi (see figure). The material is aluminum with modulus of elasticity $E = 10{,}000$ ksi and Poisson's ratio $\nu = 0.33$.

Determine the following quantities: (a) the strains for an element oriented at an angle $\theta = 30°$, (b) the principal strains, and (c) the maximum shear strains. Show the results on sketches of properly oriented elements.



Probs. 7.7-13 and 7.7-14

Solution 7.7-13 Element in plane stress

$\sigma_x = -8400$ psi $\sigma_y = 1100$ psi
$\tau_{xy} = -1700$ psi $E = 10{,}000$ ksi $\nu = 0.33$

HOOKE'S LAW (EQS. 7-34 AND 7-35)

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = -876.3 \times 10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = 387.2 \times 10^{-6}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{2\tau_{xy}(1+\nu)}{E} = -452.2 \times 10^{-6}$$

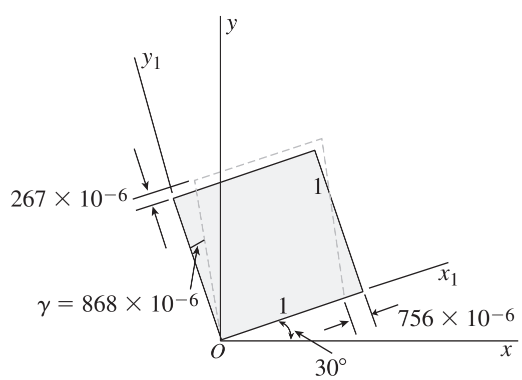
FOR $\theta = 30°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta = -756 \times 10^{-6}$$

$$\frac{\gamma_{x_1y_1}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta = 434 \times 10^{-6}$$

$$\gamma_{x_1y_1} = 868 \times 10^{-6}$$

$$\varepsilon_{y_1} = \varepsilon_x + \varepsilon_y - \varepsilon_{x_1} = 267 \times 10^{-6}$$



Principal strains

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= -245 \times 10^{-6} \pm 671 \times 10^{-6}$$

$$\varepsilon_1 = 426 \times 10^{-6} \qquad \varepsilon_2 = -916 \times 10^{-6}$$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = 0.3579$$

$$2\theta_p = 19.7^\circ \quad \text{and} \quad 199.7^\circ$$

$$\theta_p = 9.8^\circ \quad \text{and} \quad 99.8^\circ$$

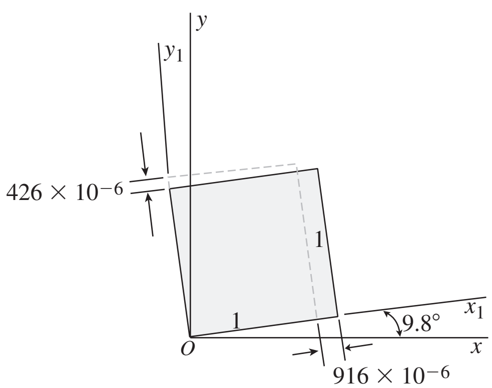
For $\theta_p = 9.8^\circ$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= -916 \times 10^{-6}$$

$$\therefore\ \theta_{p_1} = 99.8^\circ \qquad \varepsilon_1 = 426 \times 10^{-6} \quad \longleftarrow$$

$$\theta_{p_2} = 9.8^\circ \qquad \varepsilon_2 = -916 \times 10^{-6} \quad \longleftarrow$$



Maximum shear strains

$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= 671 \times 10^{-6}$$

$$\gamma_{max} = 1342 \times 10^{-6}$$

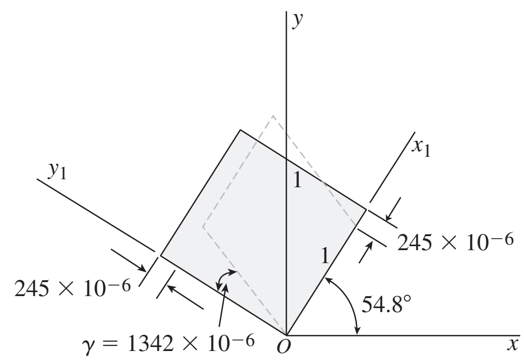
$$\theta_{s_1} = \theta_{p_1} - 45^\circ = 54.8^\circ$$

$$\gamma_{max} = 1342 \times 10^{-6} \quad \longleftarrow$$

$$\theta_{s_2} = \theta_{s_1} + 90^\circ = 144.8^\circ$$

$$\gamma_{min} = -1342 \times 10^{-6} \quad \longleftarrow$$

$$\varepsilon_{aver} = \frac{\varepsilon_x + \varepsilon_y}{2} = -245 \times 10^{-6}$$



Problem 7.7-14 Solve the preceding problem for the following data: $\sigma_x = -150$ MPa, $\sigma_y = -210$ MPa, $\tau_{xy} = -16$ MPa, and $\theta = 50^\circ$. The material is brass with $E = 100$ GPa and $\nu = 0.34$.

Solution 7.7-14 Element in plane stress

$\sigma_x = -150$ MPa $\qquad \sigma_y = -210$ MPa

$\tau_{xy} = -16$ MPa $\qquad E = 100$ GPa $\qquad \nu = 0.34$

Hooke's law (Eqs. 7-34 and 7-35)

$$\varepsilon_x = \frac{1}{E}(\sigma_x - \nu\sigma_y) = -786 \times 10^{-6}$$

$$\varepsilon_y = \frac{1}{E}(\sigma_y - \nu\sigma_x) = -1590 \times 10^{-6}$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{2\tau_{xy}(1+\nu)}{E} = -429 \times 10^{-6}$$

For $\theta = 50^\circ$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

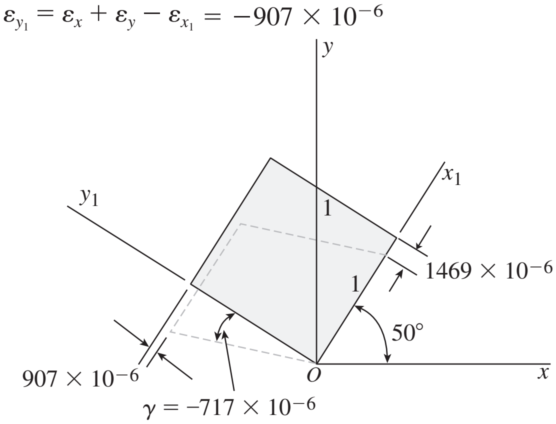
$$= -1469 \times 10^{-6}$$

$$\frac{\gamma_{x_1y_1}}{2} = -\frac{\varepsilon_x - \varepsilon_y}{2}\sin 2\theta + \frac{\gamma_{xy}}{2}\cos 2\theta$$

$$= -358.5 \times 10^{-6}$$

$$\gamma_{x_1y_1} = -717 \times 10^{-6}$$

$$\varepsilon_{y_1} = \varepsilon_x + \varepsilon_y - \varepsilon_{x_1} = -907 \times 10^{-6}$$



PRINCIPAL STRAINS

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$= -1188 \times 10^{-6} \pm 456 \times 10^{-6}$

$\varepsilon_1 = -732 \times 10^{-6}$ $\varepsilon_2 = -1644 \times 10^{-6}$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = -0.5333$$

$2\theta_p = 151.9°$ and $331.9°$

$\theta_p = 76.0°$ and $166.0°$

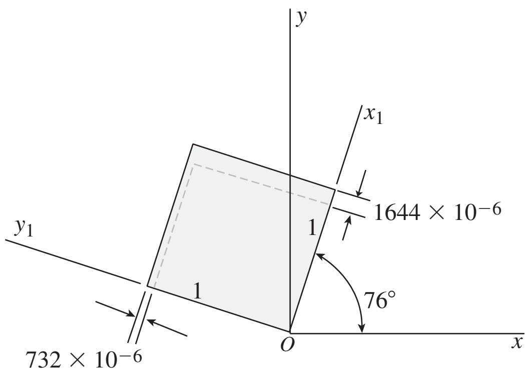
For $\theta_p = 76.0°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$= -1644 \times 10^{-6}$

$\therefore\ \theta_{p_1} = 166.0°$ $\varepsilon_1 = -732 \times 10^{-6}$ ←

$\theta_{p_2} = 76.0°$ $\varepsilon_2 = -1644 \times 10^{-6}$ ←



MAXIMUM SHEAR STRAINS

$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$= 456 \times 10^{-6}$

$\gamma_{max} = 911 \times 10^{-6}$

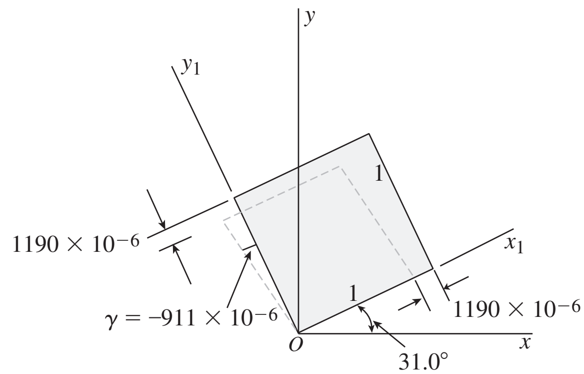
$\theta_{s_1} = \theta_{p_1} - 45° = 121.0°$

$\gamma_{max} = 911 \times 10^{-6}$ ←

$\theta_{s_2} = \theta_{s_1} - 90° = 31.0°$

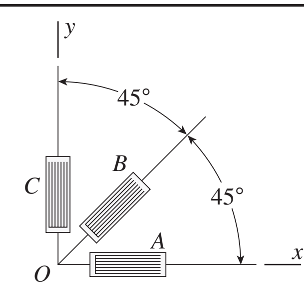
$\gamma_{min} = -911 \times 10^{-6}$ ←

$$\varepsilon_{aver} = \frac{\varepsilon_x + \varepsilon_y}{2} = -1190 \times 10^{-6}$$



Problem 7.7-15 During a test of an airplane wing, the strain gage readings from a 45° rosette (see figure) are as follows: gage A, 520×10^{-6}; gage B, 360×10^{-6}; and gage C, -80×10^{-6}.

Determine the principal strains and maximum shear strains, and show them on sketches of properly oriented elements.



Probs. 7.7-15 and 7.7-16

Solution 7.7-15 45° strain rosette

$\varepsilon_A = 520 \times 10^{-6}$ $\varepsilon_B = 360 \times 10^{-6}$

$\varepsilon_C = -80 \times 10^{-6}$

FROM EQS. (7-77) AND (7-78) OF EXAMPLE 7-8:

$\varepsilon_x = \varepsilon_A = 520 \times 10^{-6}$ $\varepsilon_y = \varepsilon_C = -80 \times 10^{-6}$

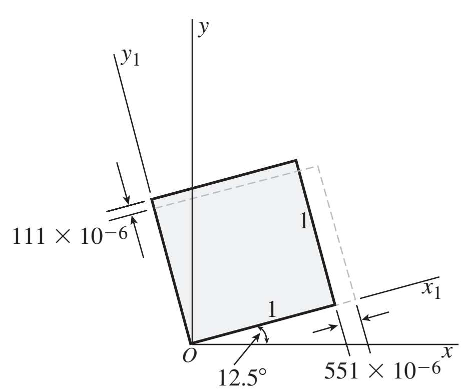
$\gamma_{xy} = 2\varepsilon_B - \varepsilon_A - \varepsilon_C = 280 \times 10^{-6}$

PRINCIPAL STRAINS

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$= 220 \times 10^{-6} \pm 331 \times 10^{-6}$

$\varepsilon_1 = 551 \times 10^{-6}$ $\varepsilon_2 = -111 \times 10^{-6}$



$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = 0.4667$$

$2\theta_p = 25.0°$ and $205.0°$

$\theta_p = 12.5°$ and $102.5°$

For $\theta_p = 12.5°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$= 551 \times 10^{-6}$

$\therefore\ \theta_{p_1} = 12.5° \quad \varepsilon_1 = 551 \times 10^{-6}$ ←

$\theta_{p_2} = 102.5° \quad \varepsilon_2 = -111 \times 10^{-6}$ ←

MAXIMUM SHEAR STRAINS

$$\frac{\gamma_{max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$= 331 \times 10^{-6}$

$\gamma_{max} = 662 \times 10^{-6}$

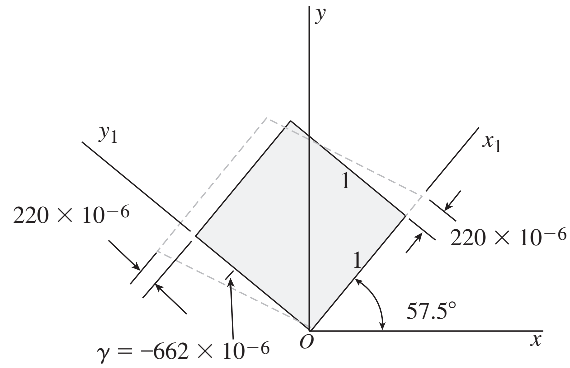
$\theta_{s_1} = \theta_{p_1} - 45° = -32.5°$ or $147.5°$

$\gamma_{max} = 662 \times 10^{-6}$ ←

$\theta_{s_2} = \theta_{s_1} + 90° = 57.5°$

$\gamma_{min} = -662 \times 10^{-6}$ ←

$$\varepsilon_{aver} = \frac{\varepsilon_x + \varepsilon_y}{2} = 220 \times 10^{-6}$$



Problem 7.7-16 A 45° strain rosette (see figure) mounted on the surface of an automobile frame gives the following readings: gage A, 310×10^{-6}; gage B, 180×10^{-6}; and gage C, -160×10^{-6}.

Determine the principal strains and maximum shear strains, and show them on sketches of properly oriented elements.

Solution 7.7-16 45° strain rosette

$\varepsilon_A = 310 \times 10^{-6} \quad \varepsilon_B = 180 \times 10^{-6}$

$\varepsilon_C = -160 \times 10^{-6}$

FROM EQS. (7-77) AND (7-78) OF EXAMPLE 7-8:

$\varepsilon_x = \varepsilon_A = 310 \times 10^{-6} \quad \varepsilon_y = \varepsilon_C = -160 \times 10^{-6}$

$\gamma_{xy} = 2\varepsilon_B - \varepsilon_A - \varepsilon_C = 210 \times 10^{-6}$

PRINCIPAL STRAINS

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$= 75 \times 10^{-6} \pm 257 \times 10^{-6}$

$\varepsilon_1 = 332 \times 10^{-6} \quad \varepsilon_2 = -182 \times 10^{-6}$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = 0.4468$$

$2\theta_p = 24.1°$ and $204.1°$

$\theta_p = 12.0°$ and $102.0°$

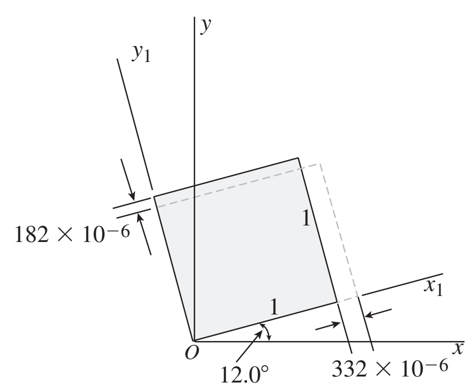
For $\theta_p = 12.0°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$= 332 \times 10^{-6}$

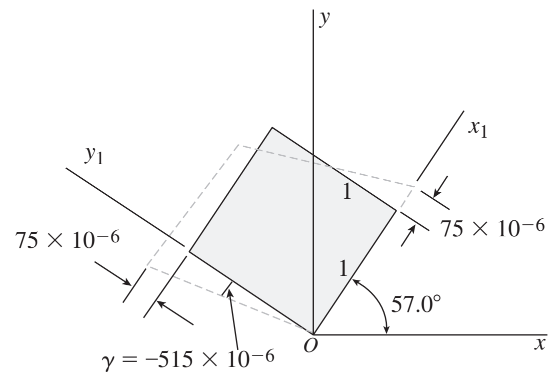
$\therefore\ \theta_{p_1} = 12.0° \quad \varepsilon_1 = 332 \times 10^{-6}$ ←

$\theta_{p_2} = 102.0° \quad \varepsilon_2 = -182 \times 10^{-6}$ ←



MAXIMUM SHEAR STRAINS

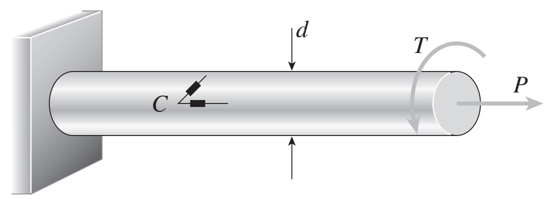
$$\frac{\gamma_{\max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$
$$= 257 \times 10^{-6}$$
$$\gamma_{\max} = 515 \times 10^{-6}$$
$$\theta_{s_1} = \theta_{p_1} - 45° = -33.0° \text{ or } 147.0°$$
$$\gamma_{\max} = 515 \times 10^{-6} \longleftarrow$$
$$\theta_{s_2} = \theta_{s_1} + 90° = 57.0°$$
$$\gamma_{\min} = -515 \times 10^{-6} \longleftarrow$$
$$\varepsilon_{\text{aver}} = \frac{\varepsilon_x + \varepsilon_y}{2} = 75 \times 10^{-6}$$

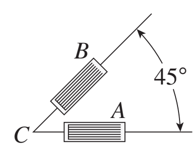


Problem 7.7-17 A solid circular bar of diameter $d = 1.5$ in. is subjected to an axial force P and a torque T (see figure). Strain gages A and B mounted on the surface of the bar give readings $\epsilon_a = 100 \times 10^{-6}$ and $\epsilon_b = -55 \times 10^{-6}$. The bar is made of steel having $E = 30 \times 10^6$ psi and $\nu = 0.29$.

(a) Determine the axial force P and the torque T.

(b) Determine the maximum shear strain $\gamma_{\max}$ and the maximum shear stress $\tau_{\max}$ in the bar.





Solution 7.7-17 Circular bar (plane stress)

Bar is subjected to a torque T and an axial force P.

$E = 30 \times 10^6$ psi $\quad \nu = 0.29$

Diameter $d = 1.5$ in.

STRAIN GAGES

At $\theta = 0°$: $\quad \varepsilon_A = \varepsilon_x = 100 \times 10^{-6}$

At $\theta = 45°$: $\quad \varepsilon_B = -55 \times 10^{-6}$

ELEMENT IN PLANE STRESS

$$\sigma_x = \frac{P}{A} = \frac{4P}{\pi d^2} \qquad \sigma_y = 0 \qquad \tau_{xy} = -\frac{16T}{\pi d^3}$$
$$\varepsilon_x = 100 \times 10^{-6} \qquad \varepsilon_y = -\nu\varepsilon_x = -29 \times 10^{-6}$$

AXIAL FORCE P

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{4P}{\pi d^2 E} \qquad P = \frac{\pi d^2 E \varepsilon_x}{4} = 5300 \text{ lb} \longleftarrow$$

SHEAR STRAIN

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = \frac{2\tau_{xy}(1+\nu)}{E} = -\frac{32T(1+\nu)}{\pi d^3 E}$$
$$= -(0.1298 \times 10^{-6})T \qquad (T = \text{lb-in.})$$

STRAIN AT $\theta = 45°$

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad (1)$$
$$\varepsilon_{x_1} = \varepsilon_B = -55 \times 10^{-6} \qquad 2\theta = 90°$$

Substitute numerical values into Eq. (1):

$$-55 \times 10^{-6} = 35.5 \times 10^{-6} - (0.0649 \times 10^{-6})T$$

Solve for T: $\quad T = 1390$ lb-in. $\longleftarrow$

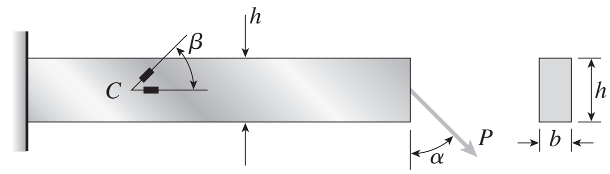
MAXIMUM SHEAR STRAIN AND MAXIMUM SHEAR STRESS

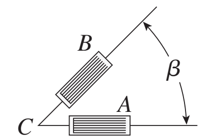
$$\gamma_{xy} = -(0.1298 \times 10^{-6})T = -180.4 \times 10^{-6} \text{ rad}$$

Eq. (7-75): $$\frac{\gamma_{\max}}{2} = \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$
$$= 111 \times 10^{-6} \text{ rad}$$
$$\gamma_{\max} = 222 \times 10^{-6} \text{ rad} \longleftarrow$$
$$\tau_{\max} = G\gamma_{\max} = 2580 \text{ psi} \longleftarrow$$

Problem 7.7-18 A cantilever beam of rectangular cross section (width $b = 25$ mm, height $h = 100$ mm) is loaded by a force P that acts at the midheight of the beam and is inclined at an angle α to the vertical (see figure). Two strain gages are placed at point C, which also is at the midheight of the beam. Gage A measures the strain in the horizontal direction and gage B measures the strain at an angle $\beta = 60°$ to the horizontal. The measured strains are $\epsilon_a = 125 \times 10^{-6}$ and $\epsilon_b = -375 \times 10^{-6}$.

Determine the force P and the angle α, assuming the material is steel with $E = 200$ GPa and $\nu = 1/3$.





Solution 7.7-18 **Cantilever beam (plane stress)**

Beam loaded by a force P acting at an angle α.

$E = 200$ GPa $\nu = 1/3$ $b = 25$ mm

$h = 100$ mm

Axial force $F = P \sin \alpha$

Shear force $V = P \cos \alpha$

(At the neutral axis, the bending moment produces no stresses.)

STRAIN GAGES

At $\theta = 0°$: $\varepsilon_A = \varepsilon_x = 125 \times 10^{-6}$

At $\theta = 60°$: $\varepsilon_B = -375 \times 10^{-6}$

ELEMENT IN PLANE STRESS

$$\sigma_x = \frac{F}{A} = \frac{P \sin\alpha}{bh} \qquad \sigma_y = 0$$

$$\tau_{xy} = -\frac{3V}{2A} = -\frac{3P\cos\alpha}{2bh}$$

$$\varepsilon_x = 125 \times 10^{-6} \qquad \varepsilon_y = -\nu\varepsilon_x = -41.67 \times 10^{-6}$$

HOOKE'S LAW

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{P\sin\alpha}{bhE}$$

$$P\sin\alpha = bhE\varepsilon_x = 62{,}500 \text{ N} \qquad (1)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = -\frac{3P\cos\alpha}{2bhG} = -\frac{3(1+\nu)P\cos\alpha}{bhE}$$

$$= -(8.0 \times 10^{-9})P\cos\alpha \qquad (2)$$

FOR $\theta = 60°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad (3)$$

$$\varepsilon_{x_1} = \varepsilon_B = -375 \times 10^{-6} \qquad 2\theta = 120°$$

Substitute into Eq. (3):

$$-375 \times 10^{-6} = 41.67 \times 10^{-6} - 41.67 \times 10^{-6} - (3.464 \times 10^{-9})P\cos\alpha$$

or $P\cos\alpha = 108{,}260$ N (4)

SOLVE EQS. (1) AND (4):

$\tan\alpha = 0.5773$ $\alpha = 30°$ ←

$P = 125$ kN ←

Problem 7.7-19 Solve the preceding problem if the cross-sectional dimensions are $b = 1.0$ in. and $h = 3.0$ in., the gage angle is $\beta = 75°$, the measured strains are $\epsilon_a = 171 \times 10^{-6}$ and $\epsilon_b = -266 \times 10^{-6}$, and the material is a magnesium alloy with modulus $E = 6.0 \times 10^6$ psi and Poisson's ratio $\nu = 0.35$.

Solution 7.7-19 Cantilever beam (plane stress)

Beam loaded by a force P acting at an angle α.
$E = 6.0 \times 10^6$ psi $\quad \nu = 0.35 \quad b = 1.0$ in.
$h = 3.0$ in.
Axial force $F = P \sin \alpha \quad$ Shear force $V = P \cos \alpha$
(At the neutral axis, the bending moment produces no stresses.)

STRAIN GAGES

At $\theta = 0°$: $\quad \varepsilon_A = \varepsilon_x = 171 \times 10^{-6}$
At $\theta = 75°$: $\quad \varepsilon_B = -266 \times 10^{-6}$

ELEMENT IN PLANE STRESS

$$\sigma_x = \frac{F}{A} = \frac{P \sin\alpha}{bh} \qquad \sigma_y = 0$$

$$\tau_{xy} = -\frac{3V}{2A} = -\frac{3P\cos\alpha}{2bh}$$

$$\varepsilon_x = 171 \times 10^{-6} \qquad \varepsilon_y = -\nu\varepsilon_x = -59.85 \times 10^{-6}$$

HOOKE'S LAW

$$\varepsilon_x = \frac{\sigma_x}{E} = \frac{P\sin\alpha}{bhE}$$

$$P\sin\alpha = bhE\varepsilon_x = 3078 \text{ lb} \qquad (1)$$

$$\gamma_{xy} = \frac{\tau_{xy}}{G} = -\frac{3P\cos\alpha}{2bhG} = -\frac{3(1+\nu)P\cos\alpha}{bhE}$$

$$= -(225.0 \times 10^{-9})P\cos\alpha \qquad (2)$$

FOR $\theta = 75°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta \qquad (3)$$

$$\varepsilon_{x_1} = \varepsilon_B = -266 \times 10^{-6} \qquad 2\theta = 150°$$

Substitute into Eq. (3):

$$-266 \times 10^{-6} = 55.575 \times 10^{-6} - 99.961 \times 10^{-6} - (56.25 \times 10^{-9})P\cos\alpha$$

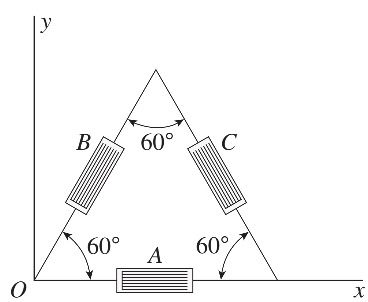
or $P\cos\alpha = 3939.8$ lb $\qquad (4)$

SOLVE EQS. (1) AND (4):

$\tan\alpha = 0.7813 \qquad \alpha = 38°$ ⟵
$P = 5000$ lb ⟵

Problem 7.7-20 A 60° strain rosette, or *delta rosette*, consists of three electrical-resistance strain gages arranged as shown in the figure. Gage A measures the normal strain ϵ_a in the direction of the x axis. Gages B and C measure the strains ϵ_b and ϵ_c in the inclined directions shown.

Obtain the equations for the strains ϵ_x, ϵ_y, and γ_{xy} associated with the xy axes.



Solution 7.7-20 Delta rosette (60° strain rosette)

STRAIN GAGES

Gage A at $\theta = 0°$ $\quad$ Strain $= \varepsilon_A$
Gage B at $\theta = 60°$ $\quad$ Strain $= \varepsilon_B$
Gage C at $\theta = 120°$ $\quad$ Strain $= \varepsilon_C$

FOR $\theta = 0°$: $\quad \varepsilon_x = \varepsilon_A$ ⟵

FOR $\theta = 60°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_B = \frac{\varepsilon_A + \varepsilon_y}{2} + \frac{\varepsilon_A - \varepsilon_y}{2}(\cos 120°) + \frac{\gamma_{xy}}{2}(\sin 120°)$$

$$\varepsilon_B = \frac{\varepsilon_A}{4} + \frac{3\varepsilon_y}{4} + \frac{\gamma_{xy}\sqrt{3}}{4} \qquad (1)$$

FOR $\theta = 120°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$\varepsilon_C = \frac{\varepsilon_A + \varepsilon_y}{2} + \frac{\varepsilon_A - \varepsilon_y}{2}(\cos 240°) + \frac{\gamma_{xy}}{2}(\sin 240°)$$

$$\varepsilon_C = \frac{\varepsilon_A}{4} + \frac{3\varepsilon_y}{4} - \frac{\gamma_{xy}\sqrt{3}}{4} \qquad (2)$$

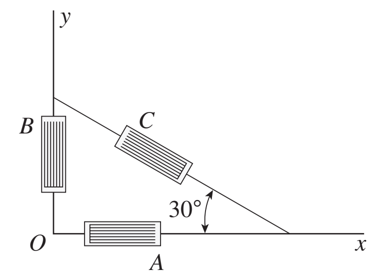
SOLVE EQS. (1) AND (2):

$$\varepsilon_y = \frac{1}{3}(2\varepsilon_B + 2\varepsilon_C - \varepsilon_A) \quad ⟵$$

$$\gamma_{xy} = \frac{2}{\sqrt{3}}(\varepsilon_B - \varepsilon_C) \quad ⟵$$

Problem 7.7-21 On the surface of a structural component in a space vehicle, the strains are monitored by means of three strain gages arranged as shown in the figure. During a certain maneuver, the following strains were recorded: $\epsilon_a = 1100 \times 10^{-6}$, $\epsilon_b = 200 \times 10^{-6}$, and $\epsilon_c = 200 \times 10^{-6}$.

Determine the principal strains and principal stresses in the material, which is a magnesium alloy for which $E = 6000$ ksi and $\nu = 0.35$. (Show the principal strains and principal stresses on sketches of properly oriented elements.)



Solution 7.7-21 **30-60-90° strain rosette**

Magnesium alloy: $E = 6000$ ksi $\quad \nu = 0.35$

STRAIN GAGES

Gage A at $\theta = 0°$ $\quad \varepsilon_A = 1100 \times 10^{-6}$
Gage B at $\theta = 90°$ $\quad \varepsilon_B = 200 \times 10^{-6}$
Gage C at $\theta = 150°$ $\quad \varepsilon_C = 200 \times 10^{-6}$

FOR $\theta = 0°$: $\quad \varepsilon_x = \varepsilon_A = 1100 \times 10^{-6}$

FOR $\theta = 90°$: $\quad \varepsilon_y = \varepsilon_B = 200 \times 10^{-6}$

FOR $\theta = 150°$:

$$\varepsilon_{x_1} = \varepsilon_C = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$200 \times 10^{-6} = 650 \times 10^{-6} + 225 \times 10^{-6} - 0.43301\gamma_{xy}$$

Solve for γ_{xy}: $\quad \gamma_{xy} = 1558.9 \times 10^{-6}$

PRINCIPAL STRAINS

$$\varepsilon_{1,2} = \frac{\varepsilon_x + \varepsilon_y}{2} \pm \sqrt{\left(\frac{\varepsilon_x - \varepsilon_y}{2}\right)^2 + \left(\frac{\gamma_{xy}}{2}\right)^2}$$

$$= 650 \times 10^{-6} \pm 900 \times 10^{-6}$$

$\varepsilon_1 = 1550 \times 10^{-6} \quad \varepsilon_2 = -250 \times 10^{-6}$

$$\tan 2\theta_p = \frac{\gamma_{xy}}{\varepsilon_x - \varepsilon_y} = \sqrt{3} = 1.7321$$

$2\theta_p = 60° \quad \theta_p = 30°$

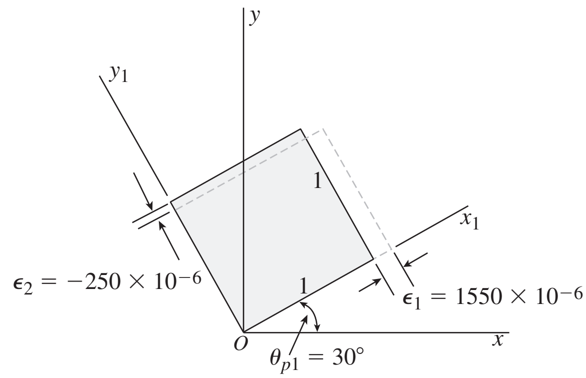
For $\theta_p = 30°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

$$= 1550 \times 10^{-6}$$

$\therefore\ \theta_{p_1} = 30° \quad \varepsilon_1 = 1550 \times 10^{-6}$ ←

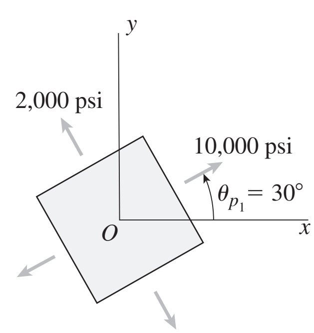
$\theta_{p_2} = 120° \quad \varepsilon_2 = -250 \times 10^{-6}$ ←



PRINCIPAL STRESSES (see Eqs. 7-36)

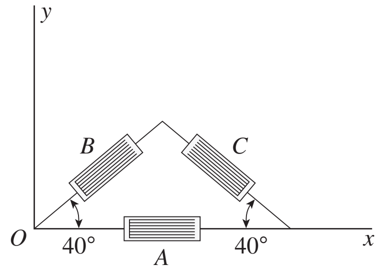
$$\sigma_1 = \frac{E}{1 - \nu^2}(\varepsilon_1 + \nu\varepsilon_2) \qquad \sigma_2 = \frac{E}{1 - \nu^2}(\varepsilon_2 + \nu\varepsilon_1)$$

Substitute numerical values:
$\sigma_1 = 10{,}000$ psi $\quad \sigma_2 = 2{,}000$ psi ←



Problem 7.7-22 The strains on the surface of an experimental device made of pure aluminum ($E = 70$ GPa, $\nu = 0.33$) and tested in a space shuttle were measured by means of strain gages. The gages were oriented as shown in the figure, and the measured strains were $\epsilon_a = 1100 \times 10^{-6}$, $\epsilon_b = 1496 \times 10^{-6}$, and $\epsilon_c = -39.44 \times 10^{-6}$.

What is the stress σ_x in the x direction?



Solution 7.7-22 40-40-100° strain rosette

Pure aluminum: $E = 70$ GPa $\quad \nu = 0.33$

STRAIN GAGES

Gage A at $\theta = 0°$ $\quad \varepsilon_A = 1100 \times 10^{-6}$
Gage B at $\theta = 40°$ $\quad \varepsilon_B = 1496 \times 10^{-6}$
Gage C at $\theta = 140°$ $\quad \varepsilon_C = -39.44 \times 10^{-6}$

FOR $\theta = 0°$: $\quad \varepsilon_x = \varepsilon_A = 1100 \times 10^{-6}$

FOR $\theta = 40°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

Substitute $\varepsilon_{x_1} = \varepsilon_B = 1496 \times 10^{-6}$ and $\varepsilon_x = 1100 \times 10^{-6}$; then simplify and rearrange:

$$0.41318\varepsilon_y + 0.49240\gamma_{xy} = 850.49 \times 10^{-6} \qquad (1)$$

FOR $\theta = 140°$:

$$\varepsilon_{x_1} = \frac{\varepsilon_x + \varepsilon_y}{2} + \frac{\varepsilon_x - \varepsilon_y}{2}\cos 2\theta + \frac{\gamma_{xy}}{2}\sin 2\theta$$

Substitute $\varepsilon_{x_1} = \varepsilon_C = -39.44 \times 10^{-6}$ and $\varepsilon_x = 1100 \times 10^{-6}$; then simplify and rearrange:

$$0.41318\varepsilon_y - 0.49240\gamma_{xy} = -684.95 \times 10^{-6} \qquad (2)$$

SOLVE EQS. (1) AND (2):

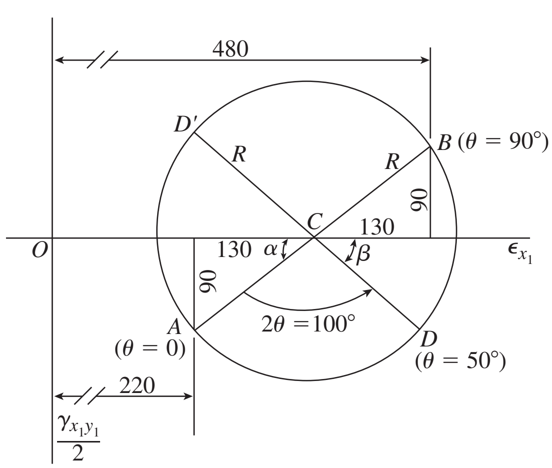
$\varepsilon_y = 200.3 \times 10^{-6} \qquad \gamma_{xy} = 1559.2 \times 10^{-6}$

HOOKE'S LAW

$$\sigma_x = \frac{E}{1 - \nu^2}(\varepsilon_x + \nu\varepsilon_y) = 91.6 \text{ MPa} \quad \leftarrow$$

Problem 7.7-23 Solve Problem 7.7-5 by using Mohr's circle for plane strain.

Solution 7.7-23 Element in plane strain



$\varepsilon_x = 220 \times 10^{-6} \qquad \varepsilon_y = 480 \times 10^{-6}$

$\gamma_{xy} = 180 \times 10^{-6} \qquad \frac{\gamma_{xy}}{2} = 90 \times 10^{-6} \qquad \theta = 50°$

$R = \sqrt{(130 \times 10^{-6})^2 + (90 \times 10^{-6})^2}$
$= 158.11 \times 10^{-6}$

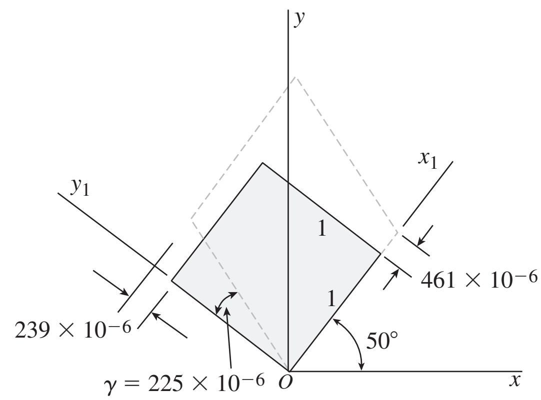
$\alpha = \arctan \dfrac{90}{130} = 34.70°$

$\beta = 180° - \alpha - 2\theta = 45.30°$

POINT C: $\varepsilon_{x_1} = 350 \times 10^{-6}$

POINT D $(\theta = 50°)$:
$\varepsilon_{x_1} = 350 \times 10^{-6} + R \cos \beta = 461 \times 10^{-6}$
$\dfrac{\gamma_{x_1y_1}}{2} = R \sin \beta = 112.4 \times 10^{-6}$
$\gamma_{x_1y_1} = 225 \times 10^{-6}$

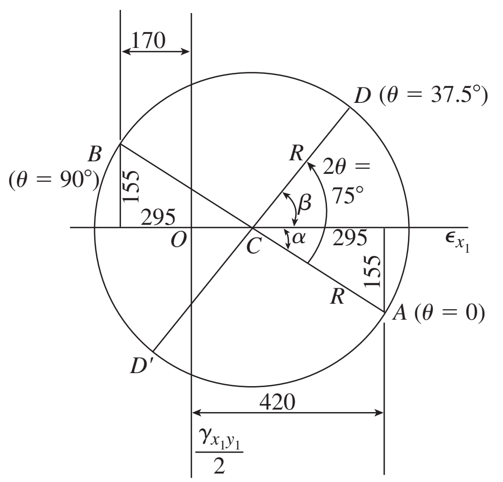
POINT D' $(\theta = 140°)$:
$\varepsilon_{x_1} = 350 \times 10^{-6} - R \cos \beta = 239 \times 10^{-6}$
$\dfrac{\gamma_{x_1y_1}}{2} = -R \sin \beta = -112.4 \times 10^{-6}$
$\gamma_{x_1y_1} = -225 \times 10^{-6}$



Problem 7.7-24 Solve Problem 7.7-6 by using Mohr's circle for plane strain.

Solution 7.7-24 Element in plane strain

$\varepsilon_x = 420 \times 10^{-6}$ $\varepsilon_y = -170 \times 10^{-6}$
$\gamma_{xy} = 310 \times 10^{-6}$ $\dfrac{\gamma_{xy}}{2} = 155 \times 10^{-6}$ $\theta = 37.5°$



$R = \sqrt{(295 \times 10^{-6})^2 + (155 \times 10^{-6})^2}$
$= 333.24 \times 10^{-6}$

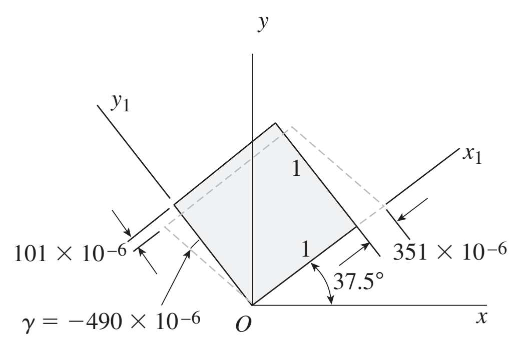
$\alpha = \arctan \dfrac{155}{295} = 27.72°$

$\beta = 2\theta - \alpha = 47.28°$

POINT C: $\varepsilon_{x_1} = 125 \times 10^{-6}$

POINT D $(\theta = 37.5°)$:
$\varepsilon_{x_1} = 125 \times 10^{-6} + R \cos \beta = 351 \times 10^{-6}$
$\dfrac{\gamma_{x_1y_1}}{2} = -R \sin \beta = -244.8 \times 10^{-6}$
$\gamma_{x_1y_1} = -490 \times 10^{-6}$

POINT D' $(\theta = 127.5°)$:
$\varepsilon_{x_1} = 125 \times 10^{-6} - R \cos \beta = -101 \times 10^{-6}$
$\dfrac{\gamma_{x_1y_1}}{2} = R \sin \beta = 244.8 \times 10^{-6}$
$\gamma_{x_1y_1} = 490 \times 10^{-6}$

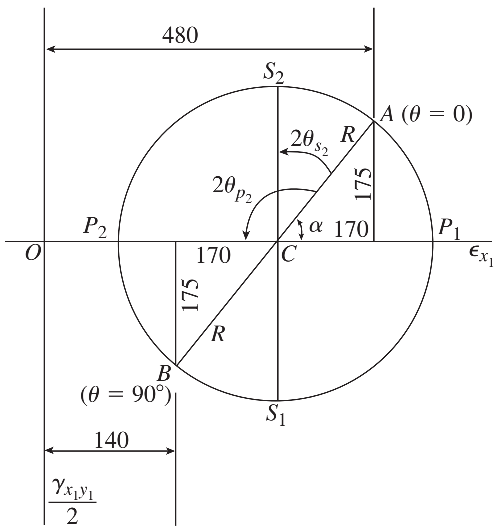


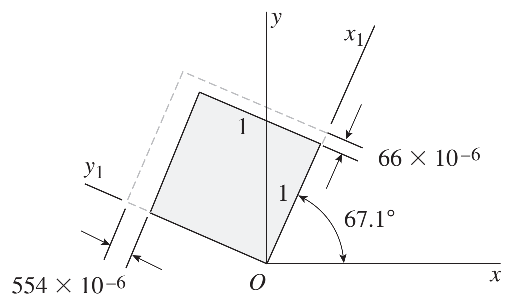
Problem 7.7-25 Solve Problem 7.7-7 by using Mohr's circle for plane strain.

Solution 7.7-25 **Element in plane strain**

$\varepsilon_x = 480 \times 10^{-6} \quad \varepsilon_y = 140 \times 10^{-6}$

$\gamma_{xy} = -350 \times 10^{-6} \quad \frac{\gamma_{xy}}{2} = -175 \times 10^{-6}$





$$R = \sqrt{(175 \times 10^{-6})^2 + (170 \times 10^{-6})^2}$$
$$= 243.98 \times 10^{-6}$$

$$\alpha = \arctan \frac{175}{170} = 45.83°$$

Point C: $\varepsilon_{x_1} = 310 \times 10^{-6}$

Principal strains

$2\theta_{p_2} = 180° - \alpha = 134.2° \quad \theta_{p_2} = 67.1°$

$2\theta_{p_1} = 2\theta_{p_2} + 180° = 314.2° \quad \theta_{p_1} = 157.1°$

Point P_1: $\varepsilon_1 = 310 \times 10^{-6} + R = 554 \times 10^{-6}$

Point P_2: $\varepsilon_2 = 310 \times 10^{-6} - R = 66 \times 10^{-6}$

Maximum shear strains

$2\theta_{s_2} = 90° - \alpha = 44.17° \quad \theta_{s_2} = 22.1°$

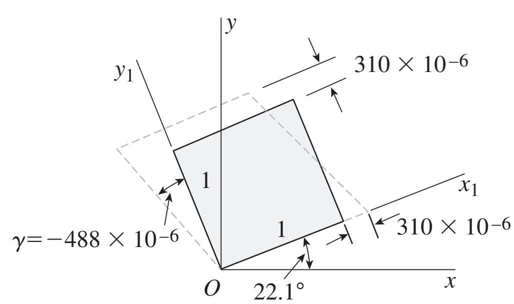
$2\theta_{s_1} = 2\theta_{s_2} + 180° = 224.17° \quad \theta_{s_1} = 112.1°$

Point S_1: $\varepsilon_{aver} = 310 \times 10^{-6}$

$\gamma_{max} = 2R = 488 \times 10^{-6}$

Point S_2: $\varepsilon_{aver} = 310 \times 10^{-6}$

$\gamma_{min} = -488 \times 10^{-6}$

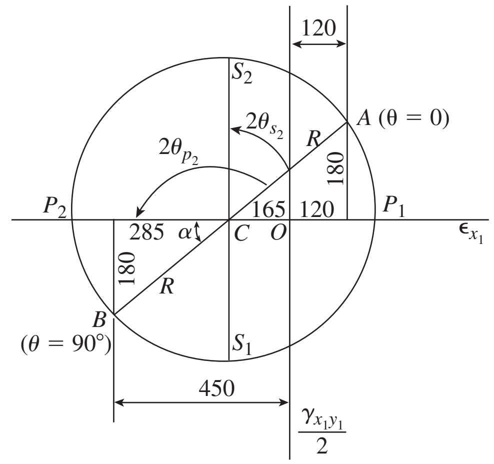


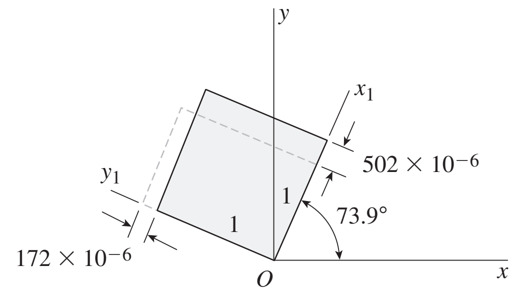
Problem 7.7-26 Solve Problem 7.7-8 by using Mohr's circle for plane strain.

Solution 7.7-26 Element in plane strain

$\varepsilon_x = 120 \times 10^{-6}$ $\quad \varepsilon_y = -450 \times 10^{-6}$

$\gamma_{xy} = -360 \times 10^{-6}$ $\quad \dfrac{\gamma_{xy}}{2} = -180 \times 10^{-6}$





$$R = \sqrt{(285 \times 10^{-6})^2 + (180 \times 10^{-6})^2}$$
$$= 337.08 \times 10^{-6}$$

$$\alpha = \arctan \frac{180}{285} = 32.28°$$

Point C: $\quad \varepsilon_{x_1} = -165 \times 10^{-6}$

Principal strains

$2\theta_{p_2} = 180° - \alpha = 147.72° \quad \theta_{p_2} = 73.9°$

$2\theta_{p_1} = 2\theta_{p_2} + 180° = 327.72° \quad \theta_{p_1} = 163.9°$

Point P_1: $\varepsilon_1 = R - 165 \times 10^{-6} = 172 \times 10^{-6}$

Point P_2: $\varepsilon_2 = -165 \times 10^{-6} - R = -502 \times 10^{-6}$

Maximum shear strains

$2\theta_{s_2} = 90° - \alpha = 57.72° \quad \theta_{s_2} = 28.9°$

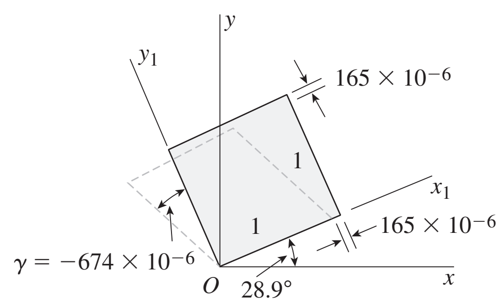
$2\theta_{s_1} = 2\theta_{s_2} + 180° = 237.72° \quad \theta_{s_1} = 118.9°$

Point S_1: $\varepsilon_{aver} = -165 \times 10^{-6}$

$\gamma_{max} = 2R = 674 \times 10^{-6}$

Point S_2: $\varepsilon_{aver} = -165 \times 10^{-6}$

$\gamma_{min} = -674 \times 10^{-6}$

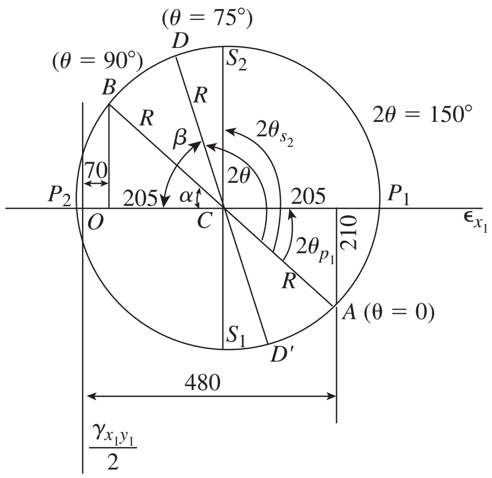


Problem 7.7-27 Solve Problem 7.7-9 by using Mohr's circle for plane strain.

Solution 7.7-27 Element in plane strain

$\varepsilon_x = 480 \times 10^{-6}$ $\quad \varepsilon_y = 70 \times 10^{-6}$

$\gamma_{xy} = 420 \times 10^{-6}$ $\quad \dfrac{\gamma_{xy}}{2} = 210 \times 10^{-6}$ $\quad \theta = 75°$



$R = \sqrt{(205 \times 10^{-6})^2 + (210 \times 10^{-6})^2}$
$= 293.47 \times 10^{-6}$

$\alpha = \arctan \dfrac{210}{205} = 45.69°$

$\beta = \alpha + 180° - 2\theta = 75.69°$

POINT C: $\varepsilon_{x_1} = 275 \times 10^{-6}$

POINT D $(\theta = 75°)$:

$\varepsilon_{x_1} = 275 \times 10^{-6} - R\cos\beta = 202 \times 10^{-6}$

$\dfrac{\gamma_{x_1y_1}}{2} = -R\sin\beta = -284.36 \times 10^{-6}$

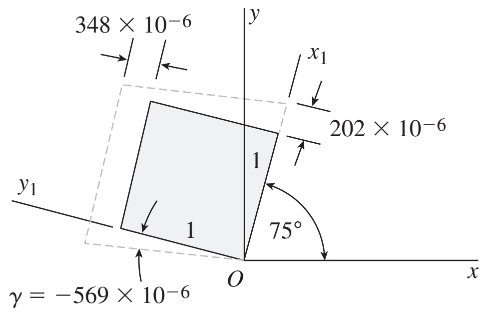
$\gamma_{x_1y_1} = -569 \times 10^{-6}$

POINT D' $(\theta = 165°)$:

$\varepsilon_{x_1} = 275 \times 10^{-6} + R\cos\beta = 348 \times 10^{-6}$

$\dfrac{\gamma_{x_1y_1}}{2} = R\sin\beta = 284.36 \times 10^{-6}$

$\gamma_{x_1y_1} = 569 \times 10^{-6}$



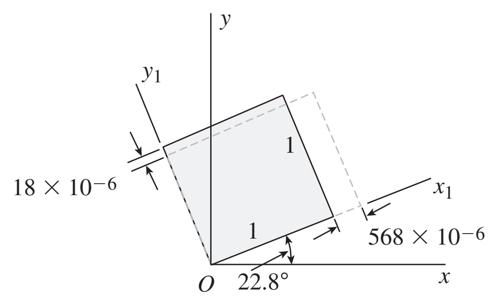
PRINCIPAL STRAINS

$2\theta_{p_1} = \alpha = 45.69°$ $\quad \theta_{p_1} = 22.8°$

$2\theta_{p_2} = 2\theta_{p_1} + 180° = 225.69°$ $\quad \theta_{p_2} = 112.8°$

Point P_1: $\varepsilon_1 = 275 \times 10^{-6} + R = 568 \times 10^{-6}$

Point P_2: $\varepsilon_2 = 275 \times 10^{-6} - R = -18 \times 10^{-6}$



MAXIMUM SHEAR STRAINS

$2\theta_{s_2} = 90° + \alpha = 135.69°$ $\quad \theta_{s_2} = 67.8°$

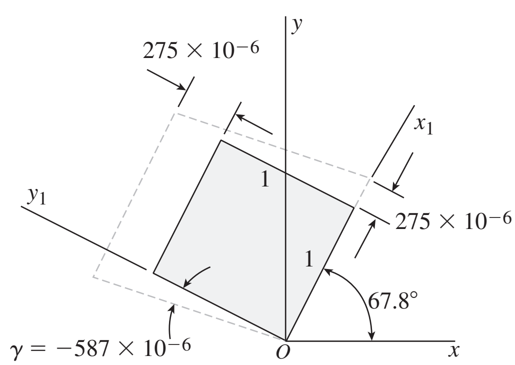
$2\theta_{s_1} = 2\theta_{s_2} + 180° = 315.69°$ $\quad \theta_{s_1} = 157.8°$

Point S_1: $\varepsilon_{aver} = 275 \times 10^{-6}$

$\gamma_{max} = 2R = 587 \times 10^{-6}$

Point S_2: $\varepsilon_{aver} = 275 \times 10^{-6}$

$\gamma_{min} = -587 \times 10^{-6}$

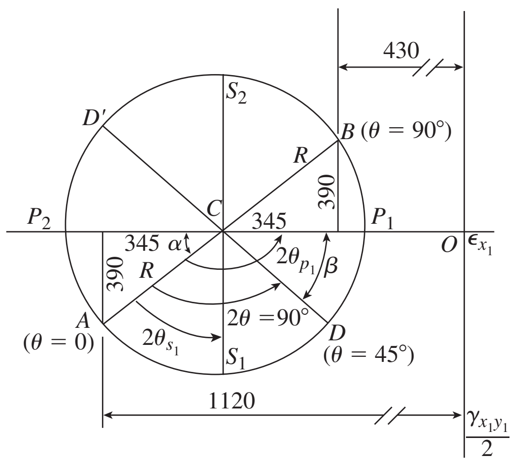


Problem 7.7-28 Solve Problem 7.7-10 by using Mohr's circle for plane strain.

Solution 7.7-28 Element in plane strain

$\varepsilon_x = -1120 \times 10^{-6} \qquad \varepsilon_y = -430 \times 10^{-6}$

$\gamma_{xy} = 780 \times 10^{-6} \qquad \dfrac{\gamma_{xy}}{2} = 390 \times 10^{-6} \qquad \theta = 45°$



$$R = \sqrt{(345 \times 10^{-6})^2 + (390 \times 10^{-6})^2}$$
$$= 520.70 \times 10^{-6}$$

$$\alpha = \arctan\frac{390}{345} = 48.50°$$

$$\beta = 180° - \alpha - 2\theta = 41.50°$$

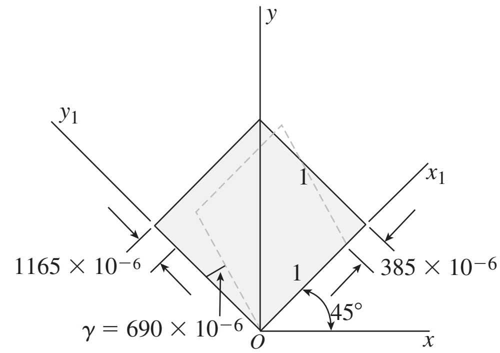
POINT C: $\quad \varepsilon_{x_1} = -775 \times 10^{-6}$

POINT D ($\theta = 45°$):

$$\varepsilon_{x_1} = -775 \times 10^{-6} + R\cos\beta = -385 \times 10^{-6}$$
$$\frac{\gamma_{x_1y_1}}{2} = R\sin\beta = 345 \times 10^{-6} \qquad \gamma_{x_1y_1} = 690 \times 10^{-6}$$

POINT D'($\theta = 135°$):

$$\varepsilon_{x_1} = -775 \times 10^{-6} - R\cos\beta = -1165 \times 10^{-6}$$
$$\frac{\gamma_{x_1y_1}}{2} = -R\sin\beta = -345 \times 10^{-6}$$
$$\gamma_{x_1y_1} = -690 \times 10^{-6}$$



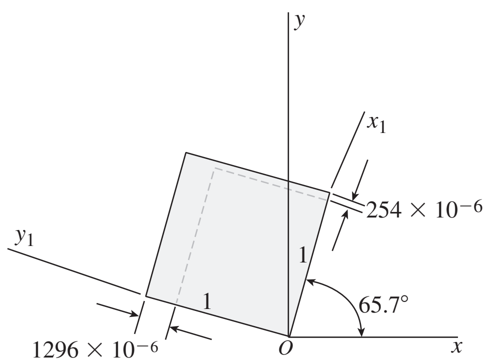
PRINCIPAL STRAINS

$2\theta_{p_1} = 180° - \alpha = 131.50° \qquad \theta_{p_1} = 65.7°$

$2\theta_{p_2} = 2\theta_{p_1} + 180° = 311.50° \qquad \theta_{p_2} = 155.7°$

Point P_1: $\varepsilon_1 = -775 \times 10^{-6} + R = -254 \times 10^{-6}$

Point P_2: $\varepsilon_2 = -775 \times 10^{-6} - R = -1296 \times 10^{-6}$



MAXIMUM SHEAR STRAINS

$2\theta_{s_1} = 90° - \alpha = 41.50° \qquad \theta_{s_1} = 20.7°$

$2\theta_{s_2} = 2\theta_{s_1} + 180° = 221.50° \qquad \theta_{s_2} = 110.7°$

Point S_1: $\varepsilon_{aver} = -775 \times 10^{-6}$

$\gamma_{max} = 2R = 1041 \times 10^{-6}$

Point S_2: $\varepsilon_{aver} = -775 \times 10^{-6}$

$\gamma_{min} = -1041 \times 10^{-6}$